Positive operators and Banach lattices
(Topics in analysis)

Marko Kandić

Outline: If the partial ordering relation on the vector space X is compatible with the vector space operations, then X is a partially ordered vector space. If a partially ordered vector space X is a lattice with respect to the partial ordering relation, then X is called a vector lattice. Most vector spaces encountered during the basic study of mathematics are vector lattices with respect to the natural partial ordering. If a norm on the Banach space X, which is also a vector lattice, satisfies $\|x\| \leq \|y\|$ for $|x| \leq |y|$, then X is a Banach lattice. Examples are $\mathbb{R}^n$ for $n \in \mathbb{N}$, $C[a,b]$ and $L^p[a,b]$ for $p \in [1,\infty]$. Among all linear operators on a given vector lattice, the positive operators play the most important role. These are the operators that map positive vectors to positive vectors.

First, we will learn about the basic properties of vector lattices and the special subspaces whose existence is ensured by ordering. In the case of Banach lattices, we will investigate the correspondence between ordering and analytic structure. A special role will be played by Banach lattices in which norm and ordering are compatible. In addition, we thoroughly investigate Banach lattices $C(K)$ and $L^p(\mu)$. We will then turn to the study of positive operators and other classes of operators on vector and Banach lattices. For certain classes of operators, we will answer the question whether a given class of operators forms a vector or a Banach lattice. We will then move on to the $C(K)$ and $L^1(\mu)$-representations of Banach lattices. We will prove that every Banach lattice is locally isomorphic to the lattice of continuous functions $C(K)$ on some compact Hausdorff space K. In the case of $L^1(\mu)$-representations, we will prove that under certain conditions a given Banach lattice is locally isomorphic to the lattice $L^1(\mu)$ for some measurable space with positive measure μ. Both representations give us additional tools to study Banach lattices.

Time permitting, we will look at the spectral theory of positive operators and the problem of invariant subspaces for positive compact operators from the order-theoretical viewpoint.

Literature:

- Y. A. Abramovich, C. D. Aliprantis, *An invitation to operator theory*, Grad. Stud. Math. **50**, American Mathematical Society, Providence, 2002.
- Y. A. Abramovich, C. D. Aliprantis, *Problems in operator theory*, Grad. Stud. Math. **51**, American Mathematical Society, Providence, 2002.
- C. D. Aliprantis, O. Burkinshaw, *Positive operators*, Springer, Dordrecht, 2006, Reprint of the 1985 original.
- W. A. J. Luxemburg, A. C. Zaanen, *Riesz Spaces I*, North-Holland Publishing Co., Amsterdam-London, 1971.

Prerequisites: The course is suitable for all PhD students and advanced MSc students with a background in functional analysis and measure theory.

Assessment: Two homework assignments and an oral exam.

Semester: Spring

Language: Slovenian or English (depending on the students in the course)