Regular Maps

Primož Potočnik and Micael Toledo

Outline: A map is usually defined as an embedding of a connected graph on a compact surface with no boundary with the property that graph divides the surface into regions homeomorphic to disks. This embedding endows the surface with additional structure, which has a somewhat geometric flavour. Indeed, maps first arose as a generalisation of convex polyhedra, which can be understood as the embeddings of a connected graph on the sphere. For instance, such an embedding of the complete graph K_4 on the sphere gives an object that is combinatorially equivalent to the tetrahedron. However, the theory of maps extends way beyond this intuitive idea, and considers surfaces that may be non-orientable or have arbitrary genus. Furthermore, during the twentieth century, interesting generalisations of the theory have arised, such as hypermaps (embeddings of hypergraphs) and maniplexes (analogs of maps in higher dimensions).

The study of maps is usually centered around the study of their symmetries. Maps that possess the highest possible degree of symmetry are called regular, and they have been extensively studied from a variety of points of view. They can be defined entirely in group theoretical terms, as permutation groups, or in purely graph-theoretical terms, as a 3-edge-coloured graph. The study of regular maps, then, takes the intuitive geometric notion of a solid and incorporates it into a theory where topology, group theory and combinatorics meet.

In this course, we will first learn about the historical foundations of the theory of regular maps and look at the various possible definitions of these objects. In the second part of the course, we will review some newer directions of research in this area.

Literature:

- P. Potočnik and M. Toledo, Regular Maps: Lecture Notes, *to appear*.
- H.S.M. Coxeter and W.O.J. Moser, Generators and Relations for Discrete Groups, 1980 (Springer).
- G. Jones and J. Wolfart, Dessins d'Enfants on Riemann Surfaces, Springer (2016).

Prerequisites: Solid understanding of basic group theory, including finitely presented groups. Basic notions from topology, including classification of closed surfaces. Basics of graph theory.

Assessment: Written and oral exam.

Semester: Summer.

Language: English