Topics in analysis: Fiber bundles

Riccardo Ugolini

Outline: The course *Fiber bundles* is aimed at Master and PhD students in Mathematics with an interest in Geometry and Analysis.

We will begin by recalling the definition of manifold and of its tangent space; the latter will be our first example of a *fibration*. In general, one can imagine a fibration over a manifold as a collection of *objects* (groups, vector spaces, topological spaces, etc.) varying smoothly (or holomorphically, in the complex case) with the points of the manifold. All the regular operations on the objects in question can be transferred to fibrations; we will speak about dual fibrations, sum of fibrations, homeomorphisms, and so on. We will conclude this first part of the course by discussing holomorphic line bundles, the Picard group, and tautological line bundles on the projective space.

An important concept is the one of *section of a fibration*, which evolves the definition of function. As we can consider collection of functions, we will also consider collections of sections; this gives rise to the theory of *sheaves*. We will review the standard De Rham cohomology under this new point of view, introducing sheaf cohomology, Cech cohomology, the abstract De Rham Theorem, Mayer-Vietoris, and applications.

If time permits, we will conclude with metrics, curvatures, and Chern characteristic classes, including vanishing theorems.

Literature:

- L. W. Tu, *An Introduction to Manifolds*, Springer (2010)
- R. Bott; L. W. Tu, *Differential Forms in Algebraic Topology*, Springer-Verlag (1982)

Prerequisites: The student should know the definition of a manifold and most of its basic properties.

Assessment: The final grade will be determined by an oral exam.

Semester: Winter

Weekly hours: 3/2

Language: English

The course will also be offered to PhD students.