Topics in algebra: Expander graphs

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Outline: Expander graphs are a type of highly connected and, at the same time, sparse graphs. They come up in numerous contexts in mathematics and computer science. Their applications span error reduction in probabilistic algorithms, sieving methods in number theory, rapid generation of pseudorandom elements in groups, graph and knot embedding problems, among others.

In this course, we will explore expander graphs through the lens of both their combinatorial as well as algebraic definition. We will look at some samples of practical applications of expander graphs and prove they exist by means of probabilistic methods. A considerable portion of the course will focus on the challenging task of constructing expander graphs explicitly. Many of these constructions are highly symmetric, derived from Lie-type groups such as $\mathrm{SL}_n(\mathbf{Z})$ and their finite quotients. We will delve into how these groups' expansive properties connect with Kazhdan's property (T) in representation theory, the concept of approximate subgroups in nonabelian additive combinatorics, and the dynamics of subset growth under multiplication in geometric group theory. Our guides through the jungle of expansion will be Kowalski and Helfgott.

Literature:

- Jean Bourgain and Alex Gamburd. "Uniform expansion bounds for Cayley graphs of $SL_2(\mathbb{F}_p)$". In: Annals of Mathematics (2008) pp. 625–642.
- Harald Andrés Helfgott. "Growth and generation in $SL_2(\mathbb{Z}/p\mathbb{Z})$". In: Annals of Mathematics (2008) pp. 601–623.
- Shlomo Hoory, Nathan Linial, and Avi Wigderson. "Expander graphs and their applications". In: Bulletin of the American Mathematical Society 43.4 (2006) pp. 439–561.
- Emmanuel Kowalski. An introduction to expander graphs. Société Mathématique de France, 2019.
- Alexander Lubotzky. "Expander graphs in pure and applied mathematics". In: Bulletin of the American Mathematical Society 49.1 (2012) pp. 113–162.
- Terence Tao. Expansion in finite simple groups of Lie type. Vol. 164. American Mathematical Soc., 2015.

Prerequisites: Basic knowledge of graph theory, group theory, probability, and functional analysis.

Assessment: Homework and oral exam at the end of the semester.

Semester: Winter

Weekly hours: 3/2

Language: English

The course will also be offered to PhD students.