Stochastic processes 3

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Outline: The path segments of a discrete-time Markov process between its successive visits to a given point are i.i.d., which is elementary to state, intuitively clear, and not too difficult to prove. The technically much more elaborate corresponding observation in continuous time is the basis of Itô's theory of excursions. The path segments – "excursions" – must here be viewed on a random time scale, the so-called inverse local time; they then become a Poisson point process: the process of excursions. Understanding the latter requires hence some familiarity with the theory of Poisson point processes, which is anyway useful and interesting in its own right. Applications of excursion theory are legion. We shall look more closely at the Brownian excursions (of Brownian motion from zero). Also, the splitting at the maximum of a Lévy process into independent pieces before an independent exponential random time can be seen to fall out naturally from Itô's excursion picture (of the Lévy process reflected in its running supremum).

Major themes: Markov processes in continuous time and space: strong Markov property, hitting times, additive functionals; Poisson point processes: compensation and exponential formula, strong renewal property; point processes of excursions: local time, inverse local time, excursion process, characteristic measure; examples: Brownian excursions, splitting at the maximum of a process with stationary indpendent increments.

Literature:

- K. Itô: Point processes attached to Markov processes, in: L. M. Le Cam, J. Neyman, E. L. Scott (editors), Berkeley Symposium on Mathematical Statistics and Probability, 1972, pp. 225-239.
- L. C. G. Rogers: A guided tour through excursions, Bulletin of the London Mathematical society, vol. 21, 1989, pp. 305-341.
- R. M. Blumenthal: Excursions of Markov Processes, Birkhäuser Boston, 1992.
- O. Kallenberg: Foundations of Modern Probability, Springer, 2002.
- P. Greenwood and J. Pitman: Fluctuation identited for Lévy processes and splitting at the maximum, Advances in Applied Probability, vol. 12, no. 4, 1980, pp. 893-902.

Prerequisites: measure-theoretic probability; basic knowledge of Markov processes, at least in a discrete setting; basic knowledge of homogeneous Poisson processes and of Brownian motion.

Assessment: oral exam

Semester: Winter

Weekly hours: 3/2

Language: Notes in English; lectures and tutorials in Slovenian or English (depending on [the preferences of] the students enrolled in the course).

The course will also be offered to PhD students.