UNIVERSITY OF LJUBLJANA
Faculty of Mathematics and Physics

# Elective courses at master's and doctoral degree study programs Department of Mathematics FMF

Academic year 2025/26

Ljubljana, 2025

Contents

# List of elective courses for 2025/26

IPXY in the course name means that the course is offered as a topics course in a given field. The descriptive/content subtitle of the course is given next to this code.

Course language:
- slo – Slovenian
- ang – English
- slo/ang – if international students are enrolled in the course, generally English
- lang + lang – the first language option refers to lectures and the second to tutorials

| Group | Course | Lecturer | Sem. | Language |
|---|---|---|---|---|
| M1 | Complex analysis | Bessonov | 2. | ang |
| | Special functions | Kostenko | 2. | ang |
| | Measure theory | Kandić | 1. | slo |
| | Operator theory | Bessonov | 1. | ang |
| M2 | IPAlg: Representation theory | Jezernik | 2. | slo + ang |
| | Combinatorics | Konvalinka | 2. | slo + ang |
| | Noncommutative algebra | Smertnig | 1. | ang |
| M3 | Algebraic topology 1 | Strle | 1. | slo/ang |
| | Algebraic topology 2 | Pavešić | 2. | slo |
| | Introduction to algebraic geometry | Šivic | 1. | slo/ang + ang |
| M4 | Numerical approximation and interpolation | Knez | 1. | slo |
| | Numerical methods for linear control systems | Plestenjak | 2. | slo/ang |
| | Numerical solving of partial differential equations | Grošelj | 2. | slo |
| M5 | Actuarial mathematics – Nonlife insurance | Hieber, Albrecher | 2. | ang |
| | Bayesian statistics | Smrekar | 2. | slo/ang + ang |
| | Time series | Basrak | 2. | ang |
| | Financial mathematics 2 | Perman | 1. | slo |
| | IPFM: Risk management | Dacorogna | 2. | ang |
| | Numerical methods for financial mathematics | Zanette | 2. | ang |
| | Probability 2 | Raič | 1. | slo |
| R1 | IPRM: Logic in computer science | Simpson | 2. | ang |
| | IPRM: Category theory | Swan | 1. | ang |
| | IPRM: Probabilistic methods in computer science | Cabello | 1. | slo/ang |
| | Cardinal arithmetic (Set theory) | Simpson | 2. | ang |
| | Mathematics with computers | Bauer | 1. | slo |
| | Computational complexity | Cabello | 2. | ang |
| | Theory of programming languages | Pretnar | 2. | slo |
| O | Astronomy | Zwitter | oba | slo |
| | Workplace experience | Žitnik, Košir | oba | slo |
| | Mathematics in industry | Knez | oba | slo |

| Group | Course | Lecturer | Sem. | Language |
|---|---|---|---|---|
| IŠRM A | Logic in computer science<br>Probabilistic methods in computer science | Simpson<br>Cabello | 2.<br>1. | ang<br>slo/ang |
| IŠRM B | IPNM: Numerical approximation and interpolation<br>IPNM: Numerical methods for linear control systems<br>IPNM: Numerical solving of partial differential equations<br>Combinatorics 2<br>Mathematics with computers<br>Computational complexity<br>Theory of programming languages | Knez<br>Plestenjak<br>Grošelj<br>Konvalinka<br>Bauer<br>Cabello<br>Pretnar | 1.<br>2.<br>2.<br>2.<br>1.<br>2.<br>2. | slo<br>slo/ang<br>slo<br>slo + ang<br>slo<br>ang<br>slo |

## List of other courses that will be offered at FMF in 2025/26 which may be chosen by Master's students

Group:
- S – general elective
- OD – extracurricular course
- DOK – PhD course

| Group | Course | Lecturer | Sem. | Language |
|---|---|---|---|---|
| S | Project work | Knez | both | slo |
| OD | Bridge<br>Voluntary work in teaching support | Drinovec Drnovšek<br>Kobal | 1.<br>both | slo<br>slo |
| DOK | IPAlg: Multiplicative Ideal Theory and Factorization Theory<br>IPAna: Hankel and Toeplitz operators<br>IPTop: Persistent homology | Smertnig<br>Bessonov<br>Virk | 2.<br>1.<br>2. | ang<br>ang<br>ang |

Below are the course content descriptions prepared by the instructors. The meaning of the labels for the weekly hours in the course descriptions is as follows:
- $p/v$ = $p$ hours of lectures and $v$ hours of tutorials per week
- $p/s/v$ = $p$ hours of lectures, $s$ hours of seminar, and $v$ hours of tutorials per week

## List of core master's courses

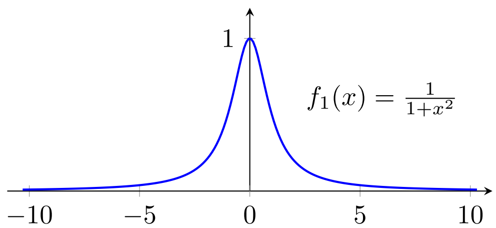
The following courses are fundamental in their groups and are as a rule offered every other year.

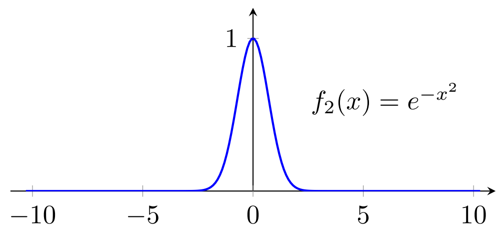
| Group | Core courses |
|---|---|
| M1 (Analysis nad mechanics) | Complex analysis<br>Partial differential equations<br>Measure theory<br>Introduction to functional analysis |
| M2 (Algebra and discrete mathematics) | Combinatorics<br>Commutative algebra<br>Noncommutative algebra<br>Graph theory |
| M3 (Geometry and topology) | Algebraic topology 1<br>Analysis on manifolds |
| M4 (Numerical mathematics) | Numerical approximation and interpolation<br>Computer aided geometric design |
| M5 (Probability, statistics and financial mathematics) | Financial mathematics 2<br>Statistics 2<br>Probability 2 |
| R1 (Computer science and mathematics) | Logic in computer science<br>Mathematics with computers<br>Computational geometry<br>Probabilistic methods in computer science |
| O (General) | Mathematical models in biology<br>Astronomy<br>Modern physics |

# Complex analysis

**Roman Bessonov**

**Outline:** Consider two functions of one real variable whose graphs are shown below:




Both functions are even, infinitely smooth and decay at infinity, their graphs looks pretty much the same. However, their Tailor's expansions

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}, \qquad e^{-x^2} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{n!},$$

have completely different behaviour: the first series converges only for $|x| < 1$, while the second series converges everywhere on $\mathbb{R}$. There is nothing special on the graph of $f_1$ near the points $x = \pm 1$, where the series for $f_1$ start to diverge. To see the real reason for the divergence, we have to extend the definition of $f_1$ to complex numbers and find the roots $z = \pm i$ of the equation $1 + z^2 = 0$ (both of them lie on the unit circle $|z| = 1$ in the complex plane).

The above example illustrates the fact that the "natural domain" of many real-variable functions is the complex plane or its open subsets. Moreover, after continuation these functions often remain differentiable with respect to the new complex variable, i.e., they are *analytic*. Analytic functions have many deep properties that differ drastically from the properties of general functions of real variable.

In the first part of the course we will collect some standard facts and instruments of the theory: differential forms, Cauchy formula, argument principle, Schwarz lemma, Riemann mapping theorem, etc. These facts are usually known from the basic analysis course. We will briefly recall their proofs and consider examples of application. In the second part we will focus on more advaced theory and prove several deep results including Beurling's factorization theorem for bounded analytic functions, Caratheodory's extension theorem in general Jordan domains and Picard's theorem on range of analytic functions.

**Literature:**


- J. B. Conway: Functions of one complex variable I, 2nd ed., New York: Springer, 1978.
- T. W. Gamelin: Complex analysis, New York: Springer, 2001.
- W. Rudin: Real and complex analysis, 3rd ed., New York: McGraw-Hill, 1987.


**Prerequisites:** There are no prerequisites.

**Assessment:** Homeworks, oral exam.

**Semester:** Spring semester.

**Weekly hours:** 2/1/2 (lectures/seminar/recitations) per week.

**Language:** English.

# Special Functions

**Aleksey Kostenko**

**Outline:** The concept of "special function" is one that has no precise definition. From a practical point of view, a special function is a function of one variable that is not one of the "elementary functions" (algebraic, trigonometric functions, the exponential, the logarithm, and functions constructed algebraically from these functions), and is a function about which one can find information in many of the books about special functions. The aim is to cover (actually, we would only be able to touch) the following topics during this course:

- The classical orthogonal polynomials (Chebyshev, Gegenbauer, Hermite, Jacobi, Laguerre, Legendre).
- Hypergeometric functions.
- Bessel functions.
- Elliptic functions.

Most of all these special functions have appeared in XVIII–XIX centuries in solutions to differential equations arising in important problems of mathematical physics. Among numerous applications of special functions, we plan to focus on representations of compact Lie groups and their applications to quantum mechanics and also on applications to nonlinear completely integrable wave equations (e.g., KdV and Toda).

**Literature:**

- G. E. Andrews, R. Askey, and R. Roy, *Special Functions*, Cambridge University Press, Cambridge, 1999.
- R. Beals and R. Wong, *Special Functions: A Graduate Text*, Cambridge University Press, Cambridge, 2010.
- N. Ja. Vilenkin and A. U. Klimyk, *Representation of Lie Groups and Special Functions, Vol. 1*, Kluwer, Dordrecht, 1991.

**Prerequisites:** Familiarity with power series, integrals, and convergence; basic knowledge of linear algebra; some familiarity with operator theory in Hilbert spaces is desirable.

**Assessment:** Written and oral exam.

**Semester:** Spring

**Weekly hours:** 2/1/2

**Language:** English

## Measure theory

**Marko Kandić**

**Outline:** Probability theory has, in the beginning, too often examined discrete events using combinatorial methods. In 1933, Kolmogorov laid the foundations of the modern approach based on measure theory. Measure, as a concept, is a generalisation of the concepts of length, area and volume to arbitrary sets.

In this course, we begin by introducing the concepts of a measurable space and a positive measure. We will examine their fundamental properties and construct the Lebesgue measure on the real line, which corresponds to the intuitive notion of interval length. Next, we will define the measurability of functions and introduce the Lebesgue integral for non-negative measurable functions. This will be extended to the broader class of absolutely integrable measurable functions. We will derive Lebesgue's very important theorems on monotone and dominated convergence, which tell when we can interchange limit process and integration. We will define the double integral and, as in Analysis 2a, ask ourselves when the double integral is equal to the iterated ones. We will answer this question with Tonelli's and Fubini's theorems. Since Lebesgue and Riemann integrals coincide in the case of a Riemannian integrable function $f\colon [a,b] \to \mathbb{R}$, Lebesgue theory gives us new tools for integrating Riemannian integrable functions.

In the sequel, we introduce the notions of real and complex measures. We will prove the Lebesgue-Radon-Nikodým theorem and the Hahn and Jordan decomposition of the real measure. We will also introduce the notion of $L^p$-spaces, which are an indispensable source of Banach spaces in functional analysis. We will then look at measures on locally compact spaces. In doing so, we will prove a theorem of Riesz concerning the representation of positive functionals on $C_c(X)$. Finally, we will look at the derivation of measures and functions.

**Literature:**

- G. B. Folland, *Real analysis: Modern techniques and their applications*, J. Wiley & Sons, New York, 1999.
- B. Magajna, *Osnove teorije mere*, DMFA Založništvo, Ljubljana, 2011.
- W. Rudin, *Real and Complex analysis*, McGraw-Hill, New York, 1987.

**Prerequisites:** Linear algebra and mathematical analysis

**Assessment:** One homework assignment instead of mid-terms, which is taken into account for the grade. The final grade is a combination of the homework (15%), the written (50%) and the theoretical exam (35%).

**Semester:** Winter

**Weekly hours:** Lectures and problem solving sessions 3/2

**Language:** Slovenian

# Operator theory

**Roman Bessonov**

**Outline:** Operator theory often provides a convenient language for the study of the most general properties of objects arising in mathematics and mathematical physics. One reason for this is the fundamental character of the notion of a linear operator, another one – the fact that operator theory is well developed and implies strong consequences under minimal assumptions.

The course has two main goals. The first goal is to introduce basic concepts and prove some fundamental theorems related to them. The second goal is to demonstrate how the general theory applies in particular situations arising in diverse fields of mathematics.

We will discuss, in particular, compact operators, invariant subspaces, fixed points, Riesz-Dunford functional calculus, adjoint operators, spectral theorem, Fredholm theory, Schatten-von Neumann classes, the trace and determinant for operators on infinitely-dimensional spaces. Examples from the theory of differential equations, ergodic theory, theory of point processes and mathematical physics will illustrate the course.

**Literature:**

- Y. A. Abramovich, C. D. Aliprantis: An invitation to operator theory, Providence: American Mathematical Society, 2002.
- Y. A. Abramovich, C. D. Aliprantis: Problems in operator theory, Providence: American Mathematical Society, 2002.
- F. Albiac, N. J. Kalton: Topics in Banach space theory, New York: Springer, 2006.
- J. B. Conway: A Course in functional analysis, 2nd ed. - New York: Springer, 1990.
- I. Gohberg, S. Goldberg, M. A. Kaashoek: Classes of linear operators. Vol. 1, Basel : Birkhäuser, 1990.
- G. K. Pedersen: Analysis now, New York: Springer, 1989.
- H. Radjavi, P. Rosenthal: Simultaneous triangularization, New York: Springer, 2000.
- I. Vidav: Linearni operatorji v Banachovih prostorih, Ljubljana: Društvo matematikov, fizikov in astronomov SR Slovenije, 1982.

**Prerequisites:** There are no prerequisites.

**Assessment:**

- Homeworks
- Oral exam

**Semester:** Winter semester.

**Weekly hours:** 3 lectures, 2 recitations per week.

**Language:** English.

## Selected Topics in Algebra – Representation Theory

**Urban Jezernik**

**Outline:** Representation theory is concerned with the *linearisation* of abstract objects, primarily groups and their actions. It is a classical, well-developed branch of mathematics with numerous applications in other sciences. Two key goals it achieves are the following:

(1) Instead of studying a given group in the abstract, we realise it in various ways by invertible matrices. Powerful tools from linear algebra then allow a more transparent investigation of its properties. We are especially interested in the simplest matrix realisations of groups.
(2) Many situations in which groups arise through their actions can be linearised, and this linear structure can be decomposed into simple components, which we understand using the previous point.

The course first establishes the foundations of representation theory (basic definitions and examples, fundamental constructions). We then "zoom in" on any concrete representation as if under a microscope, discovering that each one is built out of *cells*, and each cell of *organelles*. Next we explore the well-developed theory of finite groups (where the microscope reveals a beautiful structure via the Fourier transform), with special focus on two fundamental families of finite groups: symmetric groups and general linear groups over a finite field. This theory has many applications, of which we shall highlight some modern ones (in number theory, combinatorics, and random processes on groups). Finally, we examine several examples of representations of important families of infinite groups (compact groups and linear groups, both continuous and discrete).

**Literature:**

- U. Jezernik, *Teorija upodobitev*, lecture notes, `https://urbanjezernik.github.io/teorija-upodobitev/`, 2025.
- E. Kowalski, *An Introduction to the Representation Theory of Groups*, AMS, 2014.
- W. Fulton, J. Harris, *Representation Theory: A First Course*, Springer GTM 129, 2004.
- J. P. Serre, *Linear Representations of Finite Groups*, Springer GTM 42, 1977.

**Prerequisites:** Algebra 2 is required, Algebra 3 is recommended.

**Assessment:** Homework assignments and an oral exam.

**Semester:** Spring.

**Weekly hours:** 3/2.

**Language:** Slovene (lectures) and English (exercise classes).

## Combinatorics

**Matjaž Konvalinka**

**Outline:** Enumerative combinatorics is a field of discrete mathematics that deals with counting mathematical objects with certain properties. Problems range from very easy (e.g., the number of permutations of a set with $n$ elements is $n!$) to (probably) unsolvable (e.g., finding the number of non-isomorphic graphs at $n$ points). In this course, we will upgrade our knowledge of basic counting problems (permutations, partitions, the twelvefold way), $q$-analogs, Pólya theory, partially ordered sets and Möbius inversion. The emphasis will be on generating functions and formal power series (algebra of formal power series, calculus with generating functions, the exponential formula, Lagrange inversion) and their applications (solving recursive equations, finding averages and standard deviations, approximating counting sequences).
In the course, we will cover the vast majority of the content required for the combinatorial part of the Master's exam in Discrete Mathematics.

**Literature:**

- R. P. Stanley, Enumerative Combinatorics, Volume 1 & 2
- M. Bona, A walk through combinatorics: an introduction to enumeration and graph theory
- H. Wilf, generatingfunctionology

**Prerequisites:** Knowledge of basic principles of counting (acquired, for example, in the course Discrete Mathematics 1 in the undergraduate program Mathematics or Financial Mathematics or in the course Combinatorics in the undergraduate program I□RM).

**Assessment:** Computational exam, theoretical exam.

**Semester:** Spring

**Weekly hours:** 3 hours of lectures, 2 hours of recitations

**Language:** lectures in Slovenian, recitations in English

# Noncommutative Algebra

**Daniel Smertnig**

**Outline:** Noncommutative algebra concerns itself with the study of noncommutative rings and their modules. Typical examples of noncommutative rings are rings of operators (where the multiplication represents composition of functions), group algebras, matrix rings, and division rings. Topics include:

- noetherian rings and modules,
- semisimple rings and modules and the Wedderburn-Artin Theorem,
- the Jacobson radical and the Hopkins-Levitzki Theorem,
- primitive rings and the Jacobson Density Theorem,
- the Krull–Remak–Schmidt–Azumaya Theorem (direct-sum decomposition of modules),
- Maschke's Theorem on group algebras,
- central simple algebras.

**Literature:**

- M. Brešar. Introduction to Noncommutative Algebra, Springer, 2014.
- B. Farb, R. K. Dennis. Noncommutative Algebra, Springer, 1993.
- T. Y. Lam. A first course in noncommutative rings, Springer, 1991.
- T. Y. Lam. Lectures on modules and rings, Springer, 1999.

**Prerequisites:** Algebra 2 and 3.

**Assessment:** Oral (lecture) and written (exercises). The written assessment includes homework.

**Semester:** Winter

**Weekly hours:** 3/2

**Language:** English

# Algebraic topology 1

**Sašo Strle**

**Outline:** The goal of topology is to understand spaces up to homeomorphism, but in general this is too difficult a problem, since it cannot be solved algorithmically even for closed manifolds (of dimension at least 4). Therefore, we first distinguish spaces by coarser relations, which are further simplified with the help of algebraic invariants – numbers or more complicated algebraic objects associated with spaces, such as the winding number, Euler characteristic, fundamental group, etc.

The computation of algebraic invariants is usually simpler for spaces that are systematically built from elementary pieces. We will look at polyhedra (and the associated simplicial complexes) and CW complexes. On the other hand, most algebraic invariants of a space can also be defined for general spaces – for example, the fundamental group, which measures whether a loop in the space can be continuously deformed to a point. This gives us information about the "holes" in the space – a missing point in the plane, a missing line or a closed curve in three-dimensional space, and so on. With the fundamental group, we can, for instance, reduce the problem of classifying spaces in the first step to the problem of classifying possible fundamental groups, from which the aforementioned result about the impossibility of an effective classification of manifolds follows. At the same time, the fundamental group is closely related to the covering spaces of a given space – this relation is the topological analogue of the relation between a field extension and its Galois group. We will also study the homology groups of a space (together with the corresponding homological algebra), which, roughly speaking, measure whether a sphere in the space does or does not bound.

It turns out that algebra is not only a tool for obtaining topological information, but that the theory can also be used in the other direction to prove interesting algebraic results, for example that every subgroup of a free group is free, and that the free group $F_n$ on $n$ generators embeds as a subgroup in $F_2$.

Methods of algebraic topology are also important in applications, e.g. for recognizing characteristic properties of spaces when we only know a set of numerical data representing a sample of points in the space. This is the basis of computational topology. On the other hand, there are versions of homology that can be used to study properties of embedded subspaces, for example Khovanov homology.

**Literature:**


- A. Hatcher, *Algebraic topology*, Cambridge University Press, 2001; dostopno tudi na `https://pi.math.cornell.edu/~hatcher/AT/AT.pdf`.
- M. J. Greenberg in J. R. Harper, *Algebraic topology – A first course*, The Benjamin/Cummings publishing company, 1981.
- J. Munkres, *Elements of Algebraic Topology*, Addison-Wesley, 1984.


**Prerequisites:** Expected: General Topology and Abstract Algebra, Recommended: Introduction to Geometric Topology (quotient spaces and manifolds).

**Assessment:** Homeworks, written and oral exam.

**Semester:** Winter

**Weekly hours:** 3/2

**Language:** Slovenian or English (depending on students enrolled in the course)

## Algebraic Topology 2

**Petar Pavešić**

**Outline:** Homotopy, homotopy equivalence, extensions and liftings of homotopies, homotopy category.

Cohomology groups, definition and properties, computation, applications. Construction of cohomology groups. Cohomology ring

Homotopy groups, exact sequences of a pair and of a fibration, homotopy excision.

**Literature:**

- A. Hatcher: Algebraic Topology, Cambridge Univ. Press, Cambridge, 2002

**Prerequisites:** simplicial and CW-complexes, fundamental group, homology (basic content of Algebraic Topology 1)

**Assessment:** written and oral exam

**Semester:** second

**Weekly hours:** 2/1/2

**Language:** Slovene

# Introduction to algebraic geometry

**Klemen Šivic**

**Outline:**

The main objects that are studied at this course are solutions of polynomial equations. These sets are called varieties. On one hand, we can study their geometric properties such as dimension and singular points. On the other hand, varieties are tightly connected to ideals in polynomial rings, so we can use techniques from commutative algebra. Hence, algebraic geometry connects algebra and geometry. Consequently, the knowledge of algebraic geometry may be applied to various fields of mathematics, for example, when studying geometric objects or when solving equations. Algebraic geometry is useful also in other areas, for example in theoretical physics.

In the modern algebraic geometry working only with varieties is often insufficient. For example, there are many more commutative rings than coordinate rings of varieties. Therefore we introduce schemes as generalizations of varieties. Schemes enable us to use geometric methods in other areas, such as number theory.

The following topics will be covered in the course:

- Afine and projective varieties, Zariski topology, irreducibility.
- Hilbert's Nullstellensatz, correspondence between varieties and ideals.
- Polynomial, regular and rational maps, coordinate ring and rings of regular maps, basics about sheaves.
- Classical constructions: Segre and Veronese varieties, determinantal varieties, secant varieties, Grassmann varieties.
- Dimension of varieties, dimension of fibres of regular maps.
- Tangent space to variety, smooth points, differential of a regular map, resolution of singularitiey (blow-up).
- Spectrum of a ring, introduction to schemes.

**Literature:**


- D. Cox, J. Little, D. O'Shea: *Ideals, varieties, and algorithms*, Springer, 2007.
- A. Gathmann. *Algebraic Geometry.* Class Notes, TU Kaiserslautern, 2021/22, https://www.mathematik.uni-kl.de/ gathmann/class/alggeom-2021/alggeom-2021.pdf
- B. Hassett: *Introduction to algebraic geometry*, Cambridge Univ. Press, 2007.
- J. Harris: *Algebraic Geometry : A First Course,* Springer, 1995.
- K. Hulek: *Elementary Algebraic Geometry,* AMS, Providence, 2003.
- I. Shafarevich: *Basic Algebraic Geometry I : Varieties in Projective Space*, Springer, 1994.


**Prerequisites:** Algebra 1 and 2 from the bachelor studies of mathematics. Knowledge from Algebra 3 and Commutative algebra is helpful, but not obligatory. We will recall the needed facts from these two subjects. Knowledge of Algebraic curves may help with geometric intuition.

**Assessment:** Homeworks during the semester, oral exam at the end of semester.

**Semester:** winter

**Weekly hours:** 3 hours of lectures, 2 hours of exercise classes

**Language:** lectures in Slovene, exercise classes in English

## Numerical approximation and interpolation

**Marjeta Knez**

**Outline:** The course introduces mathematical techniques that are essential for the approximate solving of various practical problems. It introduces classes of functions suitable for approximation, such as polynomials, piecewise polynomial functions or splines, trigonometric polynomials, and others. Students become familiar with criteria that define the approximate functions. These include optimal schemes such as uniform approximation with polynomials or least squares approximation, as well as simpler linear approaches like interpolation. The course also explores various algorithms and procedures for constructing approximants and criteria for evaluating their quality. Among other topics, students are introduced to B-splines, which form a numerically stable basis for the space of piece +wise polynomial functions—a basis whose elements are non-negative, have local support, and form a partition of unity. This course provides essential knowledge for subsequent courses in numerical analysis.

**Literature:**

- J. Kozak, Numerična analiza, DMFA - založništvo, Ljubljana 2008.
- M. Knez, J. Grošelj, Numerična aproksimacija in interpolacija, zbirka nalog z rešitvami, DMFA - založništvo, Ljubljana 2020.
- S. D. Conte, C. de Boor, Elementary Numerical Analysis, McGraw Hill, New York, 1980.
- D. Kincaid, W. Cheney, Numerical Analysis, Brooks/Cole, Pacific Grove, 1996.
- C. de Boor, A Practical Guide to Splines, Springer-Verlag, New York, 2001.

**Prerequisites:** A solid background in mathematical analysis, along with a basic understanding of numerical mathematics and knowledge in using MATLAB.

**Assessment:** A written exam based on practical exercises, a theoretical exam (written or oral), MATLAB-based homework assignments, assessed through quizzes.

**Semester:** Winter

**Weekly hours:** $3/2$

**Language:** Slovenian

# Numerical method for linear control systems

**Bor Plestenjak**

**Outline:** We will study linear time-invariant control systems of the form

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t) + Du(t)$$

for $t \geq t_0$ with initial condition $x(t_0) = x_0$. In the above $A$ is the state matrix, $B$ is the input matrix, $C$ is the output matrix, $D$ is the direct transmission matrix, $x(t)$ is the state vector, $u(t)$ is the input vector, and $y(t)$ is the output vector.

The above form can be used to describe the control of an air conditioning device, an autopilot in a plane, and other dynamical systems that adapt to external data so that output matches the desired output as closely as possible (e.g., the prescribed room temperature). For this to be possible, real parts of all eigenvalues of the matrix A must be strictly negative.

The emphasis will be on efficient algorithms for matrix problems that arise in this area, but also elsewhere. Among others, we will consider numerical methods for the following problems:

- how to compute the exponential function of a matrix $M(t) = e^{At}$ (and other functions of matrices),
- how to solve the Lyapunov matrix equation $XA + A^T X = C$,
- how to solve the Sylvester matrix equation $XA + BX = C$,
- how to solve the Riccatijeve matrix equation $XA + A^T X + XBR^{-1}B^T X + Q = 0$.

**Keywwords:** linear control system, state matrix, stability, controlability, observability, open-loop and close-loop systems, Lyapunov equation, Sylvester equation, Riccati equation, system stabilization.

**Literature:**

- B. Plestenjak: Numerične metode za linearne sisteme upravljanja, skripta, 2022.
- B. Plestenjak: Numerične metode za linearne sisteme upravljanja, skripta vaj, 2022.
- P. J. Antsaklis, A. N. Michel: Linear systems, Birkhäuser, Basel, 2006.
- K. J. Åström, R. M. Murray: Feedback systems: an introduction for scientists and engineers, 2nd ed., Princeton University Press, Princeton, 2021.
- B. N. Datta: Numerical Methods for Linear Control Systems, Elsevier Academic Press, San Diego, 2004.

**Prerequisites:** mandatory numerical courses from first cycle programmes

**Assessment:** 2 homework assignments in the form of quizzes (20%), written part of the exam (40%), after a positive assessment from the total of homework assignments and the written part of the exam, an oral exam (40%) must be passed.

**Semester:** spring

**Weekly hours:** 3 hours of lectures, 2 hours of tutorial

**Language:** Slovenian or English (depending on students enrolled in the course)

## Numerical solving of partial differential equations

**Jan Grošelj**

**Outline:** The course focuses on numerical and computational methods for solving partial differential equations. It introduces students to numerical methods, their analysis, and implementation, and presents practical problems in which specific approaches are particularly effective.

The following topics will be covered: Partial differential equations. Model second-order problems. Elliptic type equations. Poisson's equation. Finite difference method. Discrete maximum principle and global error estimation. Iterative solution of discretized equations. Jacobi, Gauss–Seidel, and SOR methods. ADI method. Krylov subspace methods. Multigrid methods. Variational methods. Different types of finite element methods. Parabolic type equations. Heat conduction. Explicit and implicit numerical schemes. Crank–Nicolson method. Consistency, stability, and convergence. Hyperbolic type equations. Wave equation. Characteristics, characteristic variables. Finite difference method. Courant condition. Convergence of finite difference approximations for model problems. Method of characteristics. Basics of the radial basis function method.

The MATLAB package for basic solving of partial differential equations in two dimensions, `pdeModeler`, will also be presented.

**Literature:**

- J. Kozak, Numerična analiza, DMFA – založništvo, Ljubljana 2008.

**Prerequisites:** It is recommended to have previously completed the elective course *Numerical approximation and interpolation*. For those who have not taken this course, the lecturer will include a brief bridge in the lectures. A solid knowledge of the MATLAB programming language is also highly desirable.

**Assessment:** Homework assignments with quizzes conducted on the computer. Written exam. Oral exam.

**Semester:** Spring

**Weekly hours:** 3/2

**Language:** Slovenian

# Actuarial Mathematics – Nonlife Insurance
# Aktuarska matematika – neživljenjska zavarovanja

**Hansjoerg Albrecher and Peter Hieber, Université de Lausanne, Switzerland**

**Opis/Outline:**
Individual and collective risk model
Loss distributions
Modeling of claim frequencies
Methods to compute aggregate insurance losses
Premium principles
Reinsurance
Claims reserving
(Stochastic) Chain Ladder method
Ratemaking
Generalized linear models
Mutual insurance and risk management
Credibility theory

**Cilji/Objectives:**
The course introduces the basic concepts of non-life insurance. This includes understanding risk theory and risk management, stochastic modeling of the underlying risks, risk pooling and diversification and claims reserving.
Further, ratemaking for insurance contracts is covered, introducing premium discrimination, the impact of inflation and model calibration. The course is very practical, illustrating theoretical results by examples and practical insights.

**Pridobljene kompetence/Intended learning outcomes:**
Understanding risks involved in non-life insurance, diversification and risk quantification
Knowledge of basic stochastic models for insurance
Application of theory to real insurance examples
Understanding the main factors for pricing and ratemaking in life insurance

Transferable skills: Applying mathematical and statistical concepts in insurance.

**Literatura/Literature:**

- S. A. Klugman, H. H. Panjer, G. E. Willmot: Loss Models : From Data to Decisions, Wiley, 1998.
- R. Kaas, M. Goovaerts, J. Dhaene, M Denuit: Modern Actuarial Risk Theory, Boston, Kluwer, 2001.
- H. Albrecher, J. Beirlant, H. Teugels: Reinsurance: Actuarial and Statistical Aspects, Wiley, 2017


**Pričakovano predznanje/Prerequisites:** Introductory courses in probability and in statistics.

**Ocenjevanje/Assessment:**

**Semester:** Spring

**Tedenske ure/Weekly hours:** 2/1/2

**Jezik/Language:** English

## Bayesian statistics

**Jaka Smrekar**

**Outline:** Applications of statistics in practice typically involve the estimation of parameters based on the value of a random vector whose distribution is related to the estimated parameters. Bayesian statistics provide a means to incorporate into the estimation a prior 'opinion' about the value of the parameters in question (for example, opinion based on the results of similar estimations in the past). Even in honest frequentist problems, where there is no information from the past (or we do not wish to include it), we often have to resort to the methods of Bayesian statistics. Bayesian estimation is typically computationally intensive, and thus the usefulness of Bayesian statistics increases with increasing computer performance.

The following topics will be discussed in detail.

- Bayesian models with one and more parameters.
- Hierarchical models.
- Conjugate prior distributions.
- Algorithms for the simulation of sampling from posterior distributions.
- Bayesian regression models.

**Literature:**

- Andrew Gelman et al., Bayesian data analysis. Taylor and Francis Group, 2014.
- Peter D. Hoff, A first course in Bayesian statistical methods. Springer, 2009.

**Prerequisites:** The basics of probability. Discrete and continuous random variables and vectors and their distributions. The multivariate normal distribution and its derived distributions.

**Assessment:** Homework projects, seminar project, oral exam.

**Semester:** Winter.

**Weekly hours:** 2/1/2

**Language:** Slovenian and English

# Časovne vrste
# Time series

**Bojan Basrak, University of Zagreb, Croatia**

**Opis/Outline:** Introduction: Examples of time series. Trend and seasonality. Autocorrelation function. Multivariate normal distribution. Strong and week stationarity. Hilbert spaces and prediction. Introduction to time series modelling with R.
Stationary sequences: Linear processes. ARMA models. Causality and invertibility of ARMA processes. Infinite order MA processes. Partial autocorrelation function. Estimation of autocorrelation function and other parameters. Forecasting stationary time series. Modeling and forecasting for ARMA processes. Asymptotic behaviour of the sample mean and the autocorrelation function. Parameter estimation for ARMA processes.
Spectral analysis: Spectral density. Spectral density of ARMA processes. Herglotz theorem. Periodogram.
Nonlinear and nonstationary time series models: ARCH and GARCH models. Moments and stationary distribution of GARCH process. Exponential GARCH. ARIMA models.
Statistics for stationary process: Asymptotic results for stationary time series. Estimating trend and seasonality. Nonparametric methods.

**Literatura/Literature:**

- P.J. Brockwell, R.A. Davis. *Introduction to Time Series and Forecasting*, Springer, 2002.
- P.J. Brockwell, R.A.Davis. *Time Series: Theory and Methods*, Springer, 1991.
- W.N. Shumway, D. Stoffer, *Time Series Analysis and Its Applications*, Springer, 2006.

**Pričakovano predznanje/Prerequisites:** Introductory courses in probability and in statistics.

**Ocenjevanje/Assessment:** Written exam, seminar work and its presentation.

**Semester:** Spring

**Tedenske ure/Weekly hours:** 2/1/2

**Jezik/Language:** English

## Financial mathematics 2

**Assoc. Prof. Mihael Perman**

**Outline:** The larger part of the course is devoted to mathematical tools necessary for derivative valuation in continuous time. Once the tools are available, the course presents the main ideas with examples of real life options and their pricing.

Contents:

1. Overview of prerequisites from analysis and probability.
1.1 Functions of bounded variation.
1.2 Lebesgue-Stieltjes integral.
1.3 Convergence in $L^2$ spaces.
1.1 Maximal inequalities for discrete martingales.
2. Brownian motion.
2.1 Motivation and definition.
2.2 Markov and strong Markov property, reflection principle.
2.2 Brownian martingales.
2.3 Martingales in continuous time, quadratic variation.
2.4 Optional sampling theorem in continuous time.
3. Itô integral.
3.1 Construction, Itô isometry, properties.
3.2 Itô's lemma.
3.2 Localization and local martingales.
3.2 Stochastic integrals with respect to semimartingales.
3.2 General Itô formula.
4. Derivative pricing.
4.1 Self-financing strategies, hedging.
4.2 Black-Sholes model.
4.3 Change of measure, Girsanov theorem.
4.3 Examples of price calculations.

**Literature:**

D. Lamberton, B. Lepeyre, Introduction to Stochastic Calculus Applied to Finance, Chapman & Hall, 2000.
S. E. Shreve, Stochastic Calculus for Finance II, Continuous-Time Models, Springer, 2004.
T. Björk, Arbitrage Theory in Continuous Time, 3rd Edition, Oxford, 2009.

**Prerequisites:** Analysis: partial derivatives, parameter dependent integrals. Probability: independence, expected value, standard distributions, cconditional expectation, discrete martingales. Measure theory: Lebesgue integral, monotone and dominated convergence theorem, Fubini theorem, $L^p$ spaces. Financial mathematics: binomial pricing model, definition of derivative securities, one price principle, equivalent measures, completness of models.

**Assessment: 50% written exam, 50% seminar assignment.**

**Semester: winter**

**Weekly hours: 3/2**

**Language: Slovenian, English if there are Erasmus students.**

## Izbrana poglavja iz finančne matematike 1: Upravljanje s tveganji
## Topics in financial mathematics 1: Risk management

**Michel Dacorogna, Prime Re Solutions, Zug, Switzerland**

**Opis/Outline:** In this course, we develop the main theoretical concepts and modelling techniques of Quantitative Risk Management (QRM). The goal for the students is to acquire practical tools to solve real life problems. We discuss risk management in the context of finance and insurance, but RM applies also to other sectors of the industry. Main concepts include loss distributions, risk measures, interdependence, and concentration of (extreme) risks, techniques derived from probabilistic modelling and statistical analysis, copula and extreme value theory. We also discuss corporate finance concepts like economic valuation of liabilities, capital, capital allocation and structure of capital.

Through examples and case studies from the practice, we explain how sophisticated mathematical methods can be integrated in the efficient management of an insurance portfolio of risk. At the end, students should be able to understand how a modern financial institution manages its risks.

A. The concept of risk, risk measures, and the pricing of risk (4 hours): Definition of risk in insurance. Risk and risk measures, a coherent measure of risk. A simple example of pricing risk, what is the correct price? The various components of an insurance price. Capital to cover the risk.
B. Aggregation of risk and dependencies (4 hours): Effects of diversification on the price. The right measure of dependency. A hierarchical dependency structure to avoid over specification. Pricing within a portfolio. Dependence structure and diversification benefits.
C. Concept of capital and management of capital (4 hours): The different perspectives on capital. Risk based capital and economic capital. Capital allocation, what is the right method for what purpose. How much capital does an insurance company need? Structure of capital.
D. Designing and implementing an internal model (4 hours): History of the development of internal model. Purposes and goals of an internal model. Structure and architecture of an internal model. Model calibration and testing. Conditions for embedding the model in the business process.
E. Modelling of economic scenarios, their Impact on capital management (4 hours): The influence of the economy on an insurance company. Various ways to build economic scenario generators (ESG). The bootstrapping method to create scenarios. Yield curve modeling and stress scenarios. Testing of ESG.
F. The new Solvency Regulations and the Role of Reinsurance (4 hours): New context for the industry and new solvency regulation. Use of internal models and DFA. How to optimize a reinsurance cover. Case study: multi-lines and covers for catastrophic events.
G. Adding time diversification to risk diversification (2 hours): Bank and insurance as risk bearer and the challenges ahead. The example of natural catastrophes reserving. Measures to mitigate risk and time diversification. An investors' perspective on catastrophe risks.
H. Entreprise Risk Management (ERM), towards a holistic approach to risk management (4 hours): The context of risk management: a changing risk landscape. Risk management culture. Risk and economic capital modeling. Emerging risk management. Risk controls and processes. Strategic risk management.

**Literatura/Literature:** There is no book or article that covers the full set of chapters. The students will get a full set of slides for each chapter of the course.

**Pričakovano predznanje/Prerequisites:** Introductory courses in probability and in statistics. Students are expected to be fluent in statistical programming languages either R or Python.

**Ocenjevanje/Assessment:**

**Semester:** Spring

**Tedenske ure/Weekly hours:** 2/1/2. The lectures will be held in two week-long stays. Most of the course will be lectures, there will be some exercises and case studies discussions. A research project will be used to conclude the course. The seminar part of the course will be given by Paul Larsen.

**Jezik/Language:** English

# Numerične metode v finančni matematiki
# Numerical Methods in Financial Mathematics

**Antonino Zanette**
**University of Udine, Italy and INRIA MathRisk project, Paris, France**

**Opis/Outline:** Algorithms for option pricing in discrete models. Monte Carlo Methods for European options. Simulation methods of classical law. Inverse transform method. Computation of expectation. Variance reduction techniques. Tree methods for European and American options. Convergence orders of binomial methods. Estimating sensitivities. Numerical algorithms for portfolio insurance. Tree methods and Monte Carlo methods for Exotic options (barrier options, asian options, lookback options, rainbow options). American Monte Carlo methods. Finite difference methods for the Black-Scholes PDE equation.

**Literatura/Literature:**
- Notes, books and papers suggested by the teacher.
- J. Hull, *Options, Futures, and Other Derivatives*, Prentice Hall, 2011.
- N. H. Bingham, R. Kiesel, *Risk-Neutral Valuation: Pricing and Hedging of Financial Derivatives*, Springer Finance, 2004.
- P. Glasserman, *Monte Carlo Methods in Financial Engineering*, Springer, 2003.

**Pričakovano predznanje/Prerequisites:** It will be expected that the students are familiar with foundations of financial mathematics and numerical mathematics. It is required that they followed the course Financial Mathematics 2 (Finančna matematika 2) in the first semester or in the past.

**Ocenjevanje/Assessment:** The final examination will be composed of three parts :
- a written examination,
- an oral discussion of the topics of the course,
- a presentation of a numerical project assigned by the teacher.

**Semester:** Spring

**Tedenske ure/Weekly hours:** 2/1/2. The course will be held in a few two-day stays (3 hours of lectures per day). Other hours will be devoted to follow-up of the project development and oral discussion.

**Jezik/Language:** English

# Probability 2

**Martin Raič**

**Outline:**

**Discrete time Markov chains:** Discrete time random processes – existence and uniqueness. Markov property and transition probabilities. Stopping times, typical examples and counterexamples thereof. Strong Markov property. Markov chains with arbitrary initial state. Induced distributions, stationary distribution. Induced measures, stationary measure. Reversibility – detailed balance condition. Communication between states. Transience and recurrence, positive and null recurrence. Fundamental decomposition of state space. Asymptotic frequency of states. Characterization and existence of stationary distributions; construction of stationary measures. Periodicity of states. Limit theorems: ergodicity, strong law of large numbers. Specific results for the case of finite number of states.

**Continuous time Markov chains:** Regularity conventions. Continuous time Markov property and transition probabilities. Chapman–Kolmogorov equations, semigroup property. Stopping times, strong Markov property. Holding time, jump rate, jump transition probabilities. Embedded jump chain. Infinitesimal behavior, chain generator. Birth processes and explosion phenomenon. Existence and uniqueness, simulation of continuous time Markov chains. Communication between states. Transience and recurrence, positive and null recurrence. Stationary distribution and measure. Reversibility. Limit theorems: ergodicity, strong law of large numbers.

**Applications of Markov chains:** Waiting queue systems (birth & death system, $M/M/1$, introduction into the general theory, some important cases of waiting queue systems). Monte Carlo Markov chains (Bayesian statistics and Monte Carlo simulations, Gibbs sampler and Metropolis–Hastings algorithm, convergence of MCMC algorithms, applications in financial mathematics).

**Literature:**


- J. R. Norris: *Markov Chains.* Cambridge University Press, 1999.
- R. Durrett: *Essentials of Stochastic Processes.* Springer, New York, 2004.
- S. I. Resnick: *Adventures in Stochastic Processes.* Birkhäuser, 1992.
- L. C. G. Rogers, D. Williams: *Diffusions, Markov processes and Martingales. Vol. 1: Foundations.* $2^{\text{nd}}$ ed., Cambridge University Press, 2000.
- G. Grimmett, D. Stirzaker: *Probability and Random Processes.* $3^{\text{rd}}$ ed. Oxford University Press, 2001, 2009.


**Prerequisites:** basic probability theory, basic ordinary differential equations.

**Assessment:** Type: written exam or 2 midterm type exams, oral exam which can be partially replaced by theoretical tests. Grading: 1–5 (fail), 6–10 (pass) (according to the Statute of UL).

**Semester:** Winter

**Weekly hours:** $3/2$

**Language:** Slovenian or English (depending on students enrolled in the course). The recitations are expected to be conducted in English.

## Logic in Computer Science (Logika v računalništvu)

**Alex Simpson**

**Outline:** Applications of logic pervade computer science. This course will explore some of the variety of different logics used in computer science, looking at applications, the technology underpinning such applications, and the mathematical theory behind them.

**Literature:**

- M. Huth and M. Ryan. Logic in Computer Science: Modelling and Reasoning about Systems. Cambridge University Press. Second edition, 2004.
- K. Baier and J.-P. Katoen. Principles of Model Checking. MIT Press, 2008.
- J. Avigad, L. de Moura, S. Kong and S. Ullrich. Theorem Proving in Lean. `leanprover.github.io`.

**Prerequisites:** Basic knowledge of programming.

**Assessment:** Homeworks and oral exam.

**Semester:** Spring.

**Weekly hours:** 3/2

**Language:** English

# IPRM: Teorija kategorij

**Andrew Swan**

**Outline:** Throughout mathematics we often see examples of classes of mathematical structures together with some notion of map or *morphism* between them. For example in algebra we see rings and homomorphisms, in topology we see topological spaces and continuous maps, and in linear algebra vector spaces and linear maps. All of these are examples of *categories*. There are many notions that can be formulated and studied for categories in general, and then applied to examples throughout mathematics and computer science.

This course will cover important concepts in category theory including categories, functors, natural transformations, limits, colimits, adjunctions and monads. For each topic we will see both general definitions and theorems in category theory and their examples in mathematics and computer science.

**Literature:**

- Awodey, *Category theory*, Oxford logic guides, 2010
- Leinster, *Basic category theory*, Cambridge university press, 2014
- Mac Lane, *Categories for the working mathematician*, Graduate texts in mathematics, Springer, 1978

**Prerequisites:** None

**Assessment:** Homeworks and an oral exam

**Semester:** Winter

**Weekly hours:** 3/2

**Language:** English

**The course will also be offered to PhD students.**

## Probabilitstic Methods in Computer Science

**Sergio Cabello**

**Outline:** In this course we will encounter different uses of probability to solve algorithmic and related problems. We will introduce basic randomized algorithms and mathematically analyze their properties. Emphasis will be placed on the analysis of the expected time complexity and error probability of such algorithms.

The following topics will be considered in detail:

- Quicksort and minimum cut.
- Classes of problems and types of randomized algorithms.
- Use of polynomials in randomized algorithms.
- Chernoff bounds and their use.
- Modeling with random graphs.
- Randomized incremental algorithms and backwards analysis.
- Linear programming in low dimensions.
- Monte Carlo methods and approximate counting.
- Markov chains and their use (Metropolis algorithm)
- Hash functions.

**Prerequisites:** Basic knowledge of algorithms and (discrete) probability. Parts of the course are related to Computational Geometry and Computational Complexity, but no prior knowledge is required.

**Assessment:** written and oral exam.

**Semester:** Winter

**Weekly hours:** 3/2

**Language:** Slovenian or English (depending on students enrolled in the course)

# Set Theory (Teorija množic)

**Alex Simpson**

**Outline:** The course studies set theory as a mathematical subject in its own right, as a subject that interacts with and enriches other mathematical areas and as a foundation for mathematics as a whole. Set theory is an axiomatic theory built on a few, intuitive axioms. Astonishingly, the axioms suffice to capture all principles needed to rigorously develop the entire edifice of mathematics. Moreover, they give rise to powerful infinitary methods of proof, such as transfinite induction and Zorn's lemma, that lend themselves to mathematical applications. The same axioms also lead inexorably to the existence of bewilderingly large sets, and to mathematical questions that apparently have no definitive answer. For example, Cantor's famous *continuum hypothesis*, which can be formulated as a statement about the arithmetic of cardinal numbers, is *undecidable*, which means that it can be consistently assumed to be true, and also consistently assumed to be false.

The principal course topics are:

- The axioms of set theory.
- Set theory as a foundation of mathematics.
- Cardinality and the continuum hypothesis.
- Ordinals, transfinite induction and well-orders.
- The axiom of choice: equivalent statements and consequences.
- Alephs and cardinal arithmetic.
- Inaccessible cardinals and Grothendieck universes.
- Sets of real numbers.
- Measurable cardinals.
- Independence and consistency results.

**Literature:**

- K. Hrbacek and T. Jech. *Introduction to Set Theory, Third Edition, Revised and Expanded.* Chapman & Hall, 1999.

**Prerequisites:** General mathematical knowledge at the level of the Mathematics diploma (1-st level) programme.

**Assessment:** Homework, written exam and oral exam.

**Semester:** Spring

**Weekly hours:** 3/2

**Language:** English.

# Mathematics with Computers

**Andrej Bauer**

**Outline:**

In this course we will learn how computers are used in mathematics. This is a broad field that includes computational solutions to mathematical problems, experimental mathematics, simulations and numerical computations, visualization of mathematical objects, the use of computers in education and public outreach, and more. Each student will carry out a project on a topic of their choice and present their results.

Examples of possible project topics:

- simulations and intensive numerical computation,
- solving optimization and combinatorial problems,
- mathematical games,
- discovering patterns in collections of mathematical structures,
- visualization of mathematical objects and computer-generated art,
- projects in collaboration with industry,
- projects as part of research work.

In the first part of the course we will introduce some basic tools students will use in their projects. The central part of the course will be project-based. In the final part, students will present their projects.

**Literature:**

User manuals and documentation for the software used, depending on the individual project.

**Prerequisites:**

- basic programming skills and competent use of computers,
- willingness to learn to use new software through personal effort.

**Assessment:**

Each student will design, execute, and present a project.

**Semester:** Winter

**Weekly hours:**

The work will primarily be project-based. The lecturer and teaching assistant will monitor students' progress and provide guidance during office hours. If multiple students use the same software, a dedicated lecture will be organized. At the end of the semester, students will present their projects.

**Language:** Lectures in Slovenian; international students are welcome, as project work and presentations can be conducted in English.

## Computational Complexity

**Sergio Cabello**

**Outline:**

- Models of computation and Turing machines
- Hard problems (nondeterminism, classes P and NP, NP-completeness)
- Space complexity
- Aproximation algorithms, aproximation schemes and hardness of approximation
- Communication complexity or parameterized complexity.

**Prerequisites:** Basic knowledge of logic, algorithms, algebra, discrete mathematics and probability.

**Assessment:** written and oral exam.

**Semester:** Spring

**Weekly hours:** 3/2

**Language:** English

## Theory of Programming Languages

**Matija Pretnar**

**Outline:** Except for very specific programming languages, practically every modern language is Turing-complete, meaning that it can express all computable functions. Of course, this does not mean that all programming languages are equal. Some allow programs to be written in a much shorter and elegant way, others provide additional execution safety, and still others produce more efficient machine code, ...Most languages that perform well in all of the above areas share a well-established mathematical foundation. After all, mathematics is the most universal and elegant language.

In this course, we will learn the fundamental principles that guide the development of modern programming languages. Using examples of well-known constructs (conditional statements, functions, loops, ...), we will look at how to mathematically formalize a programming language, and use proof assistants (e.g. Lean, Agda) to demonstrate its properties.

(1) Basics of functional programming.
(2) Concrete and abstract syntax.
(3) Operational semantics.
(4) Type systems.
(5) Using proof assistants.
(6) Parametric polymorphism and type inference.
(7) Denotational semantics.
(8) Computational effects.

**Literature:**


- B. Pierce. Types and Programming Languages. Cambridge University Press, 2002.
- J. C. Reynolds. Theories of programming languages. Cambridge University Press, 1998.
- B. Pierce et al. Software Foundations. https://softwarefoundations.cis.upenn.edu/


**Prerequisites:**

- An interest in programming.
- Experience with functional programming and types is recommended, but not required.

**Assessment:** The student obtains the grade through practical homework assignments and a theoretical written exam.

**Semester:** Spring

**Weekly hours:** 3/2

**Language:** Slovenian

## Bridž
**Obštudijska dejavnost, 3 ECTS**

**Barbara Drinovec Drnovšek**

**Opis:** Bridž je družabna igra za 4 igralce oziroma 2 para, ki se igra z 52 igralnimi kartami. Sestavljena je iz dveh delov: iz licitacije in odigravanja. Pri licitaciji skušamo čim bolj natančno napovedati, koliko vzetkov bova s partnerjem dobila in katera barva bo adut. Vidimo le svoje karte, s partnerjem se sporazumevamo preko napovedi. V drugem delu igre poskuša par, ki je zmagal v licitaciji, osvojiti vsaj toliko vzetkov, kot jih je napovedal. Nasprotni par se trudi, da bi to preprečil.

Pri igri se razvija logično mišljenje, sposobnost hitrega odločanja in prilagajanje odločitev na podlagi vedno novih informacij, pa tudi socialne spretnosti in partnerski odnos.

Pri predmetu bomo spoznali osnovna pravila minibridža in bridža. Naučili se bomo osnov licitacije. Informacije, ki jih pridobimo iz licitacije, bomo uporabili pri odigravanju. Spoznali bomo temeljne prvine odigravanja: impas, ekspas, blokiranje in deblokiranje, ohranitev komunikacije, onemogočanje komunikacije med nasprotniki. Posvetili se bomo atakiranju in uporabi dovoljenih načinov komunikacije med partnerjema pri igri v obrambi.

Odigrali bomo en turnir v minibridžu in dva v bridžu.

**Pričakovano predznanje:** Vpis na prvo oziroma drugo stopnjo študijskega programa Univerze v Ljubljani. Predznanje ni potrebno.

**Ocenjevanje:** Ocenjuje se z ocenama »opravil«/»ni opravil«. Za pristop k izpitu je pogoj 75-odstotna prisotnost pri predmetu. Z dvema uspešno odigranima turnirjema študent opravi izpit.

**Semester:** zimski, ob sredah ob 16.15.

**Tedenske ure:** 1/2
Eni uri predavanj bosta sledili dve uri vaj. Študenti se bodo pridobljeno teoretično znanje naučili uporabiti v praksi.

**Jezik:** slovenski

**Ostalo:** Študent lahko obštudijsko dejavnost izbere v okviru splošnih izbirnih predmetov. Vpis v obštudijsko dejavnost bo potekal od začetka vpisov na FMF do zapolnitve prostih mest.

## Prostovoljna učna pomoč
**Obštudijska dejavnost, 3 ECTS**

**Damjan Kobal**

**Opis:** Mladi iz socialno ogroženih družin so pogosto manj uspešni pri študijskem delu in potrebujejo učno pomoč, ki pa si je ne morejo privoščiti. Po drugi strani lahko za take otroke prav učni uspeh predstavlja motivacijo in edino upanje za izhod iz negativnih socialno-družinskih ciklov.

V sodelovanju z relevantnimi humanitarnimi organizacijami, kot so na primer *Zveza prijateljev mladine Ljubljana Moste-Polje*, *Mladinski dom Malči Beličeve* ali *Slovenska filantropija*, študent izvede približno 30 ur individualnih ali skupinskih inštrukcij ali drugega spremljevalnega dela mladih v okviru organiziranih aktivnosti ustreznih humanitarnih organizacij. O vsebini in obsegu dela študent študent vodi preprost dnevnik in ob zaključku odda enostavno poročilo.

**Pričakovano predznanje:** Študentje vpisani na prvo oziroma drugo stopnjo študijskih programov Univerze v Ljubljani imajo za pričakovano delo dovolj predznanja.

**Ocenjevanje:** Ocena »opravil«/»ni opravil« se podeli na podlagi ustrezne angažiranosti.

**Semester:** zimski ali poletni

**Tedenske ure:** po dogovoru

**Jezik:** slovenski

**Ostalo:** Študent lahko obštudijsko dejavnost izbere v okviru splošnih izbirnih predmetov. Vpis v obštudijsko dejavnost bo potekal od začetka vpisov na FMF do zapolnitve prostih mest.

## Multiplicative Ideal Theory and Factorization Theory

**Daniel Smertnig**

**Outline:** In this course we study factorizations of elements into atoms (i.e., irreducible elements) in rings and monoids. We will mostly deal with the setting of commutative domains and cancellative commutative monoids. Fairly weak conditions (e.g., the ascending chain condition on principal ideals) suffice to guarantee the existence of factorizations of elements into irreducibles. In particular, noetherian domains always have such factorizations. By contrast, the uniqueness of these factorizations often fails, already in such nice rings as $\mathbb{Z}[\sqrt{-5}]$ or the ring of integer-valued polynomials Int$(\mathbb{Z})$. Factorization theory studies this non-uniqueness of factorizations by algebraic, analytic, and combinatorial means.

Topics of the course include:

- basic invariants of factorization theory (sets of lengths, elasticities, catenary degrees),
- Dedekind domains,
- Krull domains and Krull monoids,
- the divisor class group of a Krull monoid,
- monoids of zero-sum sequences,
- the transfer homorphism of factorization theory for Krull monoids,
- multiplicative ideal theory of commutative domains and monoids.

**Literature:**

- A. Geroldinger, F. Halter-Koch. Non-Unique Factorizations Algebraic, Combinatorial and Analytic Theory. CRC Press, 2006.
- F. Halter Koch (editors: A. Geroldinger and A. Reinhart). Ideal Theory of Commutative Rings and Monoids, Springer, 2025.
- F. Wang, H. Kim. Foundations of Commutative Rings and Their Modules. Springer, 2024.

**Prerequisites:** Algebra 2 and 3. Commutative algebra and number theory are helpful but not strictly required.

**Assessment:** Oral

**Semester:** Spring

**Weekly hours:** 2/0

**Language:** English

## Hankel and Toeplitz operators

**Roman Bessonov**

**Outline:** The course is an introduction to the theory of Hardy spaces and Hankel and Toeplitz operators on them. Such operators can be viewed as structured matrices, they appear in a wide variety of topics ranging from prediction theory to optimal control problems, from operator algebras to scattering theory. The course will focus on their basic properties and illustrate several important applications. Most proofs are function-theoretic in nature, lying at the edge of complex analysis and operator theory.

**Literature:**

- N.K.Nikolskii, *Operators, functions, and systems: an easy reading. Vol. 1*, Mathematical Surveys and Monographs, 92, Amer. Math. Soc., Providence, RI, 2002
- A.Böttcher and B.Silbermann, *Introduction to large truncated Toeplitz matrices*, Universitext, Springer, New York, 1999
- V.V.Peller, *Hankel operators and their applications*, Springer Monographs in Mathematics, Springer, New York, 2003
- N.K.Nikolskii, *Toeplitz matrices and operators*, Cambridge Studies in Advanced Mathematics, 182, Cambridge Univ. Press, Cambridge, 2020

**Prerequisites:** Basic complex analysis and measure theory.

**Assessment:**

- Homeworks
- Oral exam

**Semester:** Winter semester.

**Weekly hours:** 2 hours of lectures per week.

**Language:** English.

## Persistent homology

**Žiga Virk**

**Outline:** Persistent homology is a parameterised version of homology that measures the size of holes in a space. It is a driving force of topological data analysis, where it is often referred to as a stable descriptor of geometric shapes. In this course, we will present the topological, algebraic and combinatorial constructions through which persistent homology is defined. We will explain the basic principles of its computation, delve into its stability (continuity) and present some of its applications in mathematics and beyond.

**Literature:**

- Žiga Virk. *Introduction to Persistent Homology*, Založba UL FRI, University of Ljubljana, 2022.
- Herbert Edelsbrunner, John L. Harer. *Computational Topology, An Introduction*, American Mathematical Society, 2010.
- Ulrich Bauer and Michael Lesnick, *Induced Matchings and the Algebraic Stability of Persistence Barcodes*, Journal of Computational Geometry 6:2 (2015), 162–191.

**Prerequisites:**
Mandatory: fundamentals of linear algebra (Gaussian elimination, vector spaces)
Suggested: basics of algebraic topology

**Assessment:** The grade will be given based on a homework, and either a topic presentation or an oral exam.

**Semester:** Spring

**Weekly hours:** 2 hours of lectures per week

**Language:** English