FMF

UNIVERZA V LJUBLJANI
Fakulteta za matematiko in fiziko

Courses for doctoral degree study program Department of Mathematics FMF

Academic year 2025/26

Ljubljana, 2025

List of doctoral courses for 2025/26

Field	Course	Lecturer	Sem.	Lang.
Algebra	Multiplicative Ideal Theory and Factorization Theory	Smertnig	2.	ang
Analysis	Hankel and Toeplitz operators	Bessonov	1.	ang
Topology	Persistent homology	Virk	2.	ang

Below are the course content descriptions prepared by the instructors.

Multiplicative Ideal Theory and Factorization Theory

Daniel Smertnig

Outline: In this course we study factorizations of elements into atoms (i.e., irreducible elements) in rings and monoids. We will mostly deal with the setting of commutative domains and cancellative commutative monoids. Fairly weak conditions (e.g., the ascending chain condition on principal ideals) suffice to guarantee the existence of factorizations of elements into irreducibles. In particular, noetherian domains always have such factorizations. By contrast, the uniqueness of these factorizations often fails, already in such nice rings as $\mathbb{Z}[\sqrt{-5}]$ or the ring of integer-valued polynomials $\mathrm{Int}(\mathbb{Z})$. Factorization theory studies this non-uniqueness of factorizations by algebraic, analytic, and combinatorial means.

Topics of the course include:

- basic invariants of factorization theory (sets of lengths, elasticities, catenary degrees),
- Dedekind domains,
- Krull domains and Krull monoids,
- the divisor class group of a Krull monoid,
- monoids of zero-sum sequences,
- the transfer homorphism of factorization theory for Krull monoids,
- multiplicative ideal theory of commutative domains and monoids.

Literature:

- A. Geroldinger, F. Halter-Koch. Non-Unique Factorizations Algebraic, Combinatorial and Analytic Theory. CRC Press, 2006.
- F. Halter Koch (editors: A. Geroldinger and A. Reinhart). Ideal Theory of Commutative Rings and Monoids, Springer, 2025.
- F. Wang, H. Kim. Foundations of Commutative Rings and Their Modules. Springer, 2024.

Prerequisites: Algebra 2 and 3. Commutative algebra and number theory are helpful but not strictly required.

Assessment: Oral

Semester: Spring

Weekly hours: 2/0

Language: English

Hankel and Toeplitz operators

Roman Bessonov

Outline: The course is an introduction to the theory of Hardy spaces and Hankel and Toeplitz operators on them. Such operators can be viewed as structured matrices, they appear in a wide variety of topics ranging from prediction theory to optimal control problems, from operator algebras to scattering theory. The course will focus on their basic properties and illustrate several important applications. Most proofs are function-theoretic in nature, lying at the edge of complex analysis and operator theory.

Literature:

- N.K.Nikolskii, *Operators, functions, and systems: an easy reading. Vol. 1*, Mathematical Surveys and Monographs, 92, Amer. Math. Soc., Providence, RI, 2002
- A.Böttcher and B.Silbermann, *Introduction to large truncated Toeplitz matrices*, Universitext, Springer, New York, 1999
- V.V.Peller, *Hankel operators and their applications*, Springer Monographs in Mathematics, Springer, New York, 2003
- N.K.Nikolskii, *Toeplitz matrices and operators*, Cambridge Studies in Advanced Mathematics, 182, Cambridge Univ. Press, Cambridge, 2020

Prerequisites: Basic complex analysis and measure theory.

Assessment:

- Homeworks
- Oral exam

Semester: Winter semester.

Weekly hours: 2 hours of lectures per week.

Language: English.

Persistent homology

Žiga Virk

Outline: Persistent homology is a parameterised version of homology that measures the size of holes in a space. It is a driving force of topological data analysis, where it is often referred to as a stable descriptor of geometric shapes. In this course, we will present the topological, algebraic and combinatorial constructions through which persistent homology is defined. We will explain the basic principles of its computation, delve into its stability (continuity) and present some of its applications in mathematics and beyond.

Literature:

- Žiga Virk. *Introduction to Persistent Homology*, Založba UL FRI, University of Ljubljana, 2022.
- Herbert Edelsbrunner, John L. Harer. *Computational Topology, An Introduction*, American Mathematical Society, 2010.
- Ulrich Bauer and Michael Lesnick, *Induced Matchings and the Algebraic Stability of Persistence Barcodes*, Journal of Computational Geometry 6:2 (2015), 162–191.

Prerequisites:

Mandatory: fundamentals of linear algebra (Gaussian elimination, vector spaces)
Suggested: basics of algebraic topology

Assessment: The grade will be given based on a homework, and either a topic presentation or an oral exam.

Semester: Spring

Weekly hours: 2 hours of lectures per week

Language: English