

FMF | UNIVERSITY OF LJUBLJANA
Faculty of Mathematics and Physics

# Elective courses at master's and doctoral degree study programs Department of Mathematics FMF

Academic year 2026/27

Ljubljana, 2026

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# List of elective courses for 2026/27

STXY in the course name means that the course is offered as a topics course in a given field. The descriptive/content subtitle of the course is given next to this code.

Course language:
slo – Slovenian
ang – English
slo/ang – if international students are enrolled in the course, generally English

| Group | Course | Lecturer | Sem. | Language |
|---|---|---|---|---|
| M1 | Introduction to Functional Analysis | Klep | 1. | slo |
| | Introduction to Harmonic Analysis | Bessonov | 1. | ang |
| | Partial Differetial Equations | Kostenko | 2. | ang |
| M2 | Commutative Algebra | Klep | 1. | slo/ang |
| | Graph Theory | Iršič Chenoweth | 2. | slo/ang |
| | Number Theory | Smertnig | 2. | ang |
| | Selected topics of discrete mathematics: Finite automata | Kudryavtseva | 1. | slo/ang |
| M3 | Analysis on manifolds | Mrčun | 1. | slo |
| | Differential geometry | Saksida | 2. | slo/ang |
| | Topological robotics | Pavešić | 1. | slo/ang |
| M4 | Computer aided geometric design | Knez | 1. | slo |
| | Iterative numerical methods in linear algebra | Plestenjak | 2. | slo/ang |
| | Numerical integration and ordinary differential equations | Grošelj | 1. | slo |
| M5 | Actuarial mathematics – Life insurance | Marino | 2. | ang |
| | Financial mathematics 2 | Perman | 1. | slo |
| | Financial mathematics 3 | Gall | 2. | ang |
| | Optimization in finance | Agram | 2. | ang |
| | Probability 2 | Raič | 1. | slo/ang |
| | Statistics 2 | Smrekar | 1. | slo |
| | Stochastic processes 2 | Vidmar | 2. | slo/ang |
| R1 | Advanced machine learning | Todorovski | 1. | slo |
| | Computability theory | Simpson | 1. | slo/ang |
| | Computational geometry | Cabello | 1. | ang |
| | Computational Methods for Group Theory | Toledo | 1. | ang |
| | Logic | Simpson | 2. | slo/ang |
| | STOpt | Bašić | 2. | slo |
| | Symbolic computation | Smertnig | 2. | ang |
| O | Mathematics in industry | Knez | both | slo |
| | Modern physics | Grozdanov | 1. | slo/ang |
| | Workplace experience | Žitnik, Košir | both | slo |

| Group | Course | Lecturer | Sem. | Language |
|---|---|---|---|---|
| IŠRM A | Computational geometry | Cabello | 1. | ang |
| | Computer aided geometric design | Knez | 1. | slo |
| IŠRM B | Computational Methods for Group Theory | Toledo | 1. | ang |
| | Numerical integration and ordinary differential equations | Grošelj | 1. | slo |
| | Iterative numerical methods in linear algebra | Plestenjak | 2. | slo/ang |
| | Symbolic computation | Smertnig | 2. | ang |

## List of other courses that will be offered at FMF in 2026/27 which may be chosen by Master's students

Group:
- S – general elective
- OD – extracurricular course
- DOK – PhD course

| Group | Course | Lecturer | Sem. | Language |
|---|---|---|---|---|
| S | Project work | Knez | both | slo |
| OD | Bridge | Drinovec Drnovšek | 1. | slo |
| | Voluntary work in teaching support | Kobal | both | slo |
| DOK | Categorial logic | Simpson | 1. | ang |
| | Cohomological methods in complex analysis | Pavešić, Slapar | 2. | slo/ang |
| | Extension theory | Kostenko | 1. | ang |
| | Structure and randomness in graph theory and additive combinatorics | Jezernik | 2. | ang |

Below are the course content descriptions prepared by the instructors. The meaning of the labels for the weekly hours in the course descriptions is as follows:
- $p/v = p$ hours of lectures and $v$ hours of tutorials per week
- $p/s/v = p$ hours of lectures, $s$ hours of seminar, and $v$ hours of tutorials per week

## List of core master's courses

The following courses are fundamental in their groups and are as a rule offered every other year.

| Group | Core courses |
|---|---|
| M1 (Analysis nad mechanics) | Complex analysis<br>Partial differential equations<br>Measure theory<br>Introduction to functional analysis |
| M2 (Algebra and discrete mathematics) | Combinatorics<br>Commutative algebra<br>Noncommutative algebra<br>Graph theory |
| M3 (Geometry and topology) | Algebraic topology 1<br>Analysis on manifolds |
| M4 (Numerical mathematics) | Numerical approximation and interpolation<br>Computer aided geometric design |
| M5 (Probability, statistics and financial mathematics) | Financial mathematics 2<br>Statistics 2<br>Probability 2 |
| R1 (Computer science and mathematics) | Logic in computer science<br>Mathematics with computers<br>Computational geometry<br>Probabilistic methods in computer science |
| O (General) | Mathematical models in biology<br>Astronomy<br>Modern physics |

# Introduction to Functional Analysis

**Igor Klep**

**Outline:**
*Functional analysis is linear algebra in infinite dimensions.* We introduce the basic concepts of the theory of Banach spaces (= vector spaces equipped with a norm with respect to which they are complete), Hilbert spaces (= vector spaces equipped with an inner product that are complete), and linear operators between them. The simplest linear operators are linear functionals, which map from a Banach or Hilbert space to scalars. For these, we will prove the Hahn-Banach theorem and some of its consequences, e.g., in convexity. As an interesting note, we mention that in infinite-dimensional Banach or Hilbert spaces, there exist linear maps (and even linear functionals) that are not continuous.

We will also encounter compact operators, which have properties similar to matrices – operators on finite-dimensional spaces. We will prove the spectral theorem for self-adjoint compact operators and the singular value decomposition for arbitrary compact operators.

**Prerequisites:** Basics of linear algebra and analysis.

**Weekly hours:** 3/2, lectures and tutorials.

**Assessment:** During the semester, there will be two homework assignments, and at the end, a written exam. Upon successfully passing the written exam, it is also necessary to pass the theoretical part of the exam.

The final grade is determined as follows:

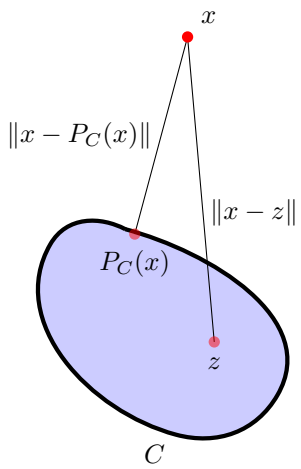
- homework assignments 8%;
- written part of the exam 50%;
- theoretical part of the exam 42%.

**Literature:**

- Peter D. Lax: *Functional Analysis*, Wiley, 2002.

**Semester:** winter

**Language:** Slovenian



## Introduction to Harmonic Analysis

**Roman Bessonov**

**Outline:** The main idea of harmonic analysis is: *everything is build from harmonics.* Originated in the the study of vibrating strings, harmonic analysis considers any function $f$ on $[-\pi, \pi]$ ("a waveform") as a sum of basic harmonics ("steady waves") $\sin kx$, $\cos kx$, with integer frequencies, i.e., in the form

$$f = \sum_{k=0}^{\infty} (a_k \sin kx + b_k \cos kx).$$

The game then is to relate various properties of $f$ to the properties of its Fourier coefficients $a_k$, $b_k$, and vice versa. Each particular choice of the set of harmonics and the class of functions $f$ for consideration corresponds to a particular branch of the theory. This is a very interesting, wide and active area of mathematics, with applications in mathematical physics and engineering. The course aims to collect the most striking results of harmonic analysis that demonstrate its power in various settings. We will meet such pearls of the theory as:

- Fourier series in general Hilbert spaces
- Paley-Wiener Theorem
- Kadec 1/4 theorem
- Uncertainty principle
- Riesz theorem on analytic measures
- Kolmogorov theorem on $(1, 1)$–weak type of the Hilbert transform
- Riesz-Thorin interpolation theorem
- Maximal functions and maximal operators on doubling metric spaces
- Atomic Hardy space and BMO
- Random stationary sequences and their prediction

A lecture or two will be devoted to each subject. Very often different subjects can be treated independently.

**Literature:**

- E.M.Stein, *Harmonic Analysis: real-variable methods, orthogonality, and oscillatory integrals.* Princeton University Press, 2016.
- N.K.Nikolskii, *Operators, functions, and systems: an easy reading. Vol. 1*, Mathematical Surveys and Monographs, 92, Amer. Math. Soc., Providence, RI, 2002

**Prerequisites:** Functional analysis and/or operator theory, measure theory, basic complex analysis.

**Assessment:** Homeworks and oral exam.

**Semester:** Winter semester.

**Weekly hours:** 3 lectures, 2 recitations per week.

**Language:** English.

**The course will also be offered to PhD students.**

## Partial Differential Equations

**Aleksey Kostenko**

**Outline:** In the course we plan to cover some important theoretical chapters as well as some nontrivial applications of differential equations. The first part of the course begins with the study of basic existence and uniqueness questions (e.g., the Cauchy–Kovalevskaya theorem, Levy's example). Then we proceed with the study of the most important 2nd order linear equations: Laplace, heat, and wave equations. The second part will be an introduction to the functional analytic treatment of partial differential equations (e.g., Sobolev spaces, distributions, linear PDEs with constant coefficients, second-order elliptic PDEs, etc.).

**Literature:**

- W. Craig, *A Course on Partial Differential Equations*, Amer. Math. Soc., 2018.
- L. C. Evans, *Partial Differential Equations*, 2nd edn., Amer. Math. Soc., 2010.
- D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, 2nd edn., Springer, 2001.
- F. John, *Partial Differential Equations*, 3rd edn., Springer, 1978.

**Prerequisites:** Basics of real and complex analysis, basics of ODEs. Some knowledge of measure theory (e.g., Lebesgue integral, $L^p$ spaces) is desirable but not necessary.

**Assessment:** The written part of the exam consists of solving homework independently, which students will receive during the semester, and the final oral exam.

**Semester:** Summer

**Weekly hours:** 3/2

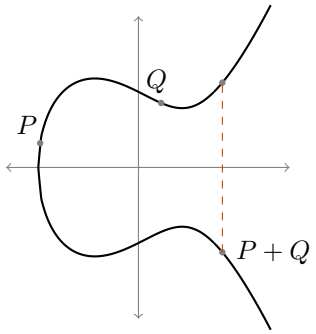
**Language:** English

# Commutative Algebra

**Igor Klep**

**Outline:**
Commutative algebra studies commutative rings, their ideals, and modules. Algebraic geometry and modern number theory are based on commutative algebra. Basic examples of commutative rings are polynomial rings, rings of algebraic integers (e.g., $\mathbb{Z}$ or $\mathbb{Z}[i]$), and $p$-adic integers. In this course, we will build upon the basic knowledge of ring theory with the following topics:

- Commutative rings, ideals, modules.
- Noetherian rings and Hilbert's basis theorem.
- Gröbner bases, basics of computing with ideals.
- Spectrum of a ring and radicals. Localization and completion.
- Affine varieties and the Zariski topology. Hilbert's Nullstellensatz.
- Morphisms, polynomial and rational maps. Isomorphisms and birationality.
- Projective varieties.
- Dimension and smoothness.
- Schemes.




$y^2 = x^3 - x + 1$

**Literature:**

- D. Cox, J. Little, D. O'Shea: Ideals, Varieties and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra, 2nd edition, Springer, New York, 2005.
- D. Eisenbud. Commutative Algebra. With a View toward Algebraic Geometry, Springer, New York, 1995.
- A. Gathmann: Commutative Algebra, lecture notes, 2013-14. `https://agag-gathmann.math.rptu.de/en/commalg.php`

**Prerequisites:** Knowledge from the course Algebra 2 is required, while knowledge of Algebra 3 is also desirable.

**Weekly hours:** 3/2. Lectures and tutorials.
Instead of midterms, homework assignments are required to qualify for the exam. Exam.

**Semester:** winter

**Language:** Slovenian or English.

## Graph theory

**Vesna Iršič Chenoweth**

**Outline:**

- Matchings and factors (maximum and perfect matchings; stable matchings; Tutte theorem; matchings in bipartite graphs; independent sets and coverings, Gallai theorem)
- Connectivity (2-connected graphs, ear decomposition; $k$-connectedness, Menger theorem; connectivity of digraphs, Menger theorem for digraphs)
- Graph colorings (bounds for chromatic number, Brooks theorem; structure of $k$-cromatic graphs, Turan theorem; chordal and perfect graphs; line graphs and edge colorings)
- Planar graphs (basic facts and results; dual graphs and triangulations; Kuratowski theorem and convex embeddings; coloring planar graphs and crossing number)
- Domination in graphs (bounds on the domination number; variations of domination; domination in Cartesian products and Vizing's conjecture)

**Literature:**


- A. Bondy, U.S.R. Murty: Graph Theory, Springer, Berlin, 2008.
- R. Diestel, Graph Theory, Springer, Berlin, 2005.
- D. West: Introduction to Graph Theory, Prentice Hall, Upper Saddle River, 2005.


**Prerequisites:** Knowledge of the basics of graph theory and combinatorics as covered in the course Discrete Mathematics 1 in the undergraduate program Mathematics.

**Assessment:** Computational exam and theoretical exam.

**Semester:** Spring

**Weekly hours:** 3/2

**Language:** Slovenian or English (depending on students enrolled in the course)

# Number Theory

**Daniel Smertnig**

**Outline:**

Number theory is one of the classical branches of mathematics, concerned with the arithmetic of integers and their generalizations. Algebraic number theory investigates number fields (finite extensions of $\mathbb{Q}$) and their rings of integers, such as $\mathbb{Z}[i]$ or $\mathbb{Z}[\zeta]$ with $\zeta$ a root of unity, using tools from abstract algebra. In fact, many concepts from ring theory and Galois theory have their historical roots in number theory. Beyond algebra, the prime number theorem provides a deep application of complex analysis, while Minkowski's geometry of numbers brings a geometric perspective that underlies two of the main finiteness results of algebraic number theory.

Concepts from number theory form the basis of many computationally hard problems that are applied in modern cryptography, for instance, rings of integers in cyclotomic fields are central to the design ofseveral post-quantum cryptography schemes standardized in recent years.

We will develop the basic notions of algebraic number theory, with a minor appearence of analytic methods in form of the prime number theorem. In particular, we cover the following topics:

- prime number theorem;
- number fields, rings of algebraic integers, discriminants, traces and norms, integral bases, and fractional ideals as well as the related algebraic notion of Dedekind domains.
- the two major finiteness theorems: finiteness of the class group and Dirichlet's unit theorem (both via the geometry of numbers);
- failure of unique factorization and its remedy via ideals;
- the decomposition of primes in number fields.

Quadratic and cyclotomic number fields serve as examples throughout.

**Literature:**

- Fröhlich, A., Taylor, M. J. *Algebraic number theory*, Cambridge University Press, 1990.
- Marcus, D. A. *Number Fields*, Springer, 2018 (2nd ed).
- Neukirch, J. *Algebraic number theory*, transl. from the German by Norbert Schappacher, Springer, 1999.

**Prerequisites:**

Algebra 1 and 2 are required. Algebra 3 is recommended but not strictly required, as long as you are willing to learn the basic results on field extensions and Galois theory on your own (they show up later in the course), as we will only recall them briefly. Basic complex analysis is used for the prime number theorem (minor part of the course).

**Assessment:** Oral (theory) and written (exercises).

**Semester:** Spring

**Weekly hours:** 3/2

**Language:** English

## Selected topics of discrete mathematics: Finite automata

**Ganna Kudryavtseva**

**Outline:**

Automata theory studies abstract models of systems whose behavior is based on transitions between states, and develops methods for the description, analysis, and application of such systems. The field brings together methods from discrete mathematics, algebra, combinatorics, logic, and theoretical computer science.

The course offers an introduction to the theory of finite automata and rational languages, with an emphasis on connections to algebra and logic. We will also treat Mealy automata which transform words (rather than merely accept or reject them) and their applications in algebra. Towards the end of the course, we will discuss several more recent results and directions of research in the area.

**Main topics of the course:**

- Finite automata, recognizable and rational languages; Kleene's theorem on the equivalence of rationality and recognizability.
- The syntactic monoid of a finite automaton. The Myhill–Nerode theorem: a language is recognizable if and only if it has a finite syntactic monoid.
- Eilenberg's theorem on the bijective correspondence between varieties of rational languages and varieties of finite monoids.
- Two special cases of Eilenberg's correspondence: Schützenberger's theorem on star-free languages and Simon's theorem on piecewise testable languages.
- Connections between finite automata and logic: Büchi's theorem on the equivalence between finite automata and monadic second-order logic.
- Automata that transform words: Mealy machines and the semigroups and groups generated by them.

**Literature:**

1. M. V. Lawson, *Finite automata*, CRC Press, 2003.
2. J.-E. Pin, *Mathematical foundations of automata theory*, 2025, available online.
3. J.-E. Pin, *Varieties of formal languages*, North Oxford Academic Publ., 1986.

**Prerequisites:** Familiarity with the basic notions of semigroup and group theory (Algebra 2) is desirable; the necessary concepts will also be briefly reviewed during the course.

**Assessment:** Homework assignments (2), a written exam combining theory and exercises, followed by an oral exam.

**Semester:** Winter

**Weekly hours:** 3/2 lectures and recitations

**Language:** Slovenian or English (depending on students enrolled in the course).

## Analysis on manifolds

**Janez Mrčun**

**Outline:** Manifolds of dimension $n$ are topological spaces that locally look as the space $\mathbb{R}^n$, but their global properties can be more involved. The unit sphere in $\mathbb{R}^3$ is an example of a manifold of dimension 2. Just like on the sphere, we can select a local coordinate system on a neighbourhood of any point of the manifold, while a global coordinate system on the whole manifold often does not exist. If we select local coordinate systems on neighbourhoods of all points of the manifold such that the transition mappings between them are smooth, we obtain a *smooth manifold*. Projective spaces and all closed matrix groups, such as SU(2) and SO(3), are examples of smooth manifolds.

Many classical quantities, well-known from differential calculus of several variables, are transformed in a natural way when changing the coordinate system, and are therefore, in a certain sense, independent of the choice of the coordinate system and can be studied global on any smooth manifold. In this way we have smooth functions, tangent spaces and tangent bundle, vector fields, tensor fields and differential forms on any smooth manifold. Under certain conditions differential forms can be integrated, and with the exterior derivative of differential forms we can calculate de Rham's cohomology of the manifold, which gives us information about the global topological properties and is a homotopy invariant of the manifold.

Smooth manifolds occur and are used in many fields of science, especially, of course, in geometry and theoretical physics. Riemannian manifolds, Lie groups, homogeneous spaces and foliations are smooth manifolds with additional geometric structure. Tangent and cotangent bundles on smooth manifolds are used to describe the configuration and the phase spaces of physical systems. A differential equation can be understood as the geometric problem of finding cross-sections of suitable vector bundles, which allows us to treat its symmetry and conservation laws in a more systematic way. Often, the local solvability of the problem is easy, but the question whether we can assemble local solutions into a global solution on the whole manifold is more complicated. For example, it is fairly easy to determine whether a vector field is locally conservative but somewhat more difficult whether it has a global potential. Indirectly, this question is answered by de Rham's cohomology.

**Literature:**

- John M. Lee: Introduction to Smooth Manifolds, 2nd edition, Springer, New York, 2013.

**Prerequisites:** analysis of functions of several real variables, linear algebra, general topology

**Assessment:** homework and oral exam

**Semester:** Winter

**Weekly hours:** 3/2

**Language:** Slovenian

# Differential geometry

**Pavle Saksida**

**Outline:** Differential geometry as we know it today has arisen from studying Riemannian manifolds. A Riemannian manifold $M$ is a smooth (differentiable) manifold, endowed with a structure which enables us to measure lengths of curves on $M$. Such a structure is called a metric.

The oldest problem in differential geometry is finding the shortest curve connecting two points on $M$. Such curves are called geodesic curves or geodesics. Geodesics are solutions of a second-order ordinary differential equation. This equation is determined by the metric tensor $g$ on $M$. A metric also yields a first-order differential operator $\nabla^g$ which assigns the vector field $Z = \nabla^g_W V$ to every pair $V$, $W$ of vector fields on $M$. Let $\dot\gamma(t)$ denote the velocity field of some curve $\gamma(t)$ on $M$. The curve $\gamma$ is a geodesic, if its velocity field solves the equation $\nabla^g_{\dot\gamma(t)}\dot\gamma(t) = 0$.

The operator $\nabla^g$ is called a covariant operator. Using $\nabla^g$ we can calculate another fundamental object of differential geometry: the Riemann curvature $R$ of $M$. When $M$ is a two-dimensional manifold, $R$ roughly corresponds to our intuitive understanding of curvature. In the case where $M$ is a four-dimensional manifold endowed with a *pseudo* Riemannian metric $g$, the operator $\nabla^g$ yields the curvature from Einstein's general relativity theory.

One can define a covariant derivative in very general contexts, without referring to a particular metric. These generalised covariant derivatives yield a more general notion of the curvature. They are the most important tool of modern differential geometry, and they are also very important in algebraic geometry, in the topology of manifolds and in many areas of mathematical physics. Some classical examples of the use of covariant derivatives are the Foucault pendulum, the explanation of why falling cats always land on their feet, the Aharonov-Bohm effect, and many others.

In the course, students will learn the fundamental theory of the covariant derivative. The introductory chapters will be dedicated to the essentials of the theory of manifolds, vector bundles and of principal bundles. The core of the course will be devoted to the theory of covariant derivatives. The general theory will then be applied to the essentials of Riemannian geometry. Time permitting, we shall describe the construction of Chern classes. These are important topological invariants, constructed using covariant derivatives.

**Literature:**

- Loring W. Tu, *Differential Geometry*, Springer 2017
- S. Kobayashi , K. Nomizu, *Foundations of Differential Geometry, vol.1*, Wiley, 1996
- P. Saksida, *Diferencialna geometrija*, https://users.fmf.uni-lj.si/saksida/DifGeomC1.pdf

**Prerequisites:** A solid understanding of the elementary theory of ordinary and partial differential equations is needed. Knowledge of topics, covered by a typical undergraduate course in differential geometry, is useful but not essential. Attending the course Analysis on manifolds is very helpful.

**Assessment:** Written exam: two sets of homework problems. Oral exam after the end of the course.

**Semester:** Spring

**Weekly hours:** 3/2

**Language:** Slovenian or English (depending on students enrolled in the course)

## Topological robotics

**Petar Pavešić**

**Outline:** We will present certain problems in robotics that can be treated by topological methods. We will describe the following basic concepts: configuration space and working space of a robot; kinematic and inverse kinematic maps and their singularities; motion planning in robotics, topological complexity of motion and manipulation planning; homotopy invariance of topological complexity, upper and lower bounds for complexity.

**Literature:**


- M. Farber, Invitation to topological robotics, European Mathematical Society, 2008.
- M. Farber, J. Gonzalez, Topology and AI, Topological Aspects of Algorithms for Autonomus Motion, EMS Series in industrial and applied mathematics, 2024.
- P. Pavešić, A Topologist's View of Kinematic Maps and Manipulation Complexity, Contemp. Math. 702 (2018), 61–83.
- A. Hatcher, Algebraic Topology, Cambridge University Press 2002.


**Prerequisites:** Some of the topics will require methods that are usually treated in algebraic topology courses (basic constructions in homotopy theory, (co)homology). Whenever necessary, these topics will be briefly recalled during the course.

**Assessment:** Depending on the specific problem, the students will prepare and present a short project or a seminar.

**Semester:** winter

**Weekly hours:** 3/2

**Language:** Slovenian or English (depending on students enrolled in the course)

## Computer aided geometric design

**Marjetka Knez**

**Outline:** Computer Aided Geometric Design (CAGD) is concerned with the mathematical description of objects for the use in numerical approximation of curves and surfaces, numerical analysis and computer graphics. It is a mathematical discipline focused on developing and studying mathematical methods for representing and working with curves, surfaces and three-dimensional objects with the aim to use these representations in various engineering fields. The methods of CAGD serve as the foundation for Computer Aided Design (CAD), Computer Aided Manufacturing (CAM) and other industrial applications like product design, robot control, manufacturing, modeling and simulations.

The program of the course is the following. First we will introduce and study Bézier curves, which represent the basic tool for describing one-dimensional objects with parametric polynomial curves. Their main properties and different algorithms for working with these curves will be presented. Further, we will consider the Bézier spline curves, the concept of geometric continuity and rational Bézier curves. The knowledge from the curve construction will be generalized to parametric surfaces, where we will focus on studying tensor product Bézier surfaces and triangular Bézier patches. Shortly we will mention also the motion design construction and the construction of curves and surfaces using subdivision schemes. The obtained theoretical knowledge will be at exercises illustrated with numerical examples. For the implementation of algorithms Matlab software package will be used.

**Literature:**


- G. Farin: Curves and surfaces for CAGD, A Practical Guide, 5th ed., Morgan Kaufmann, 2002.
- G. Farin, J. Hoschek, M.-Soo Kim: Handbook of Computer Aided Geometric Design, Amsterdam [etc.]: Elsevier, cop. 2002.
- M. J. Lai, L. Schumaker: Spline functions on triangulations, Cambridge: Cambridge University Press, 2007.


**Prerequisites:** Basic knowledge of numerical mathematics and software package Matlab.

**Assessment:** The assessment will consist of three parts:

- Two quizzes. The results of the quizzes contribute 30% to the final grade.
- A seminar paper with a presentation that contributes 35%.
- Theoretical exam that contributes 35%.

**Semester:** Winter

**Weekly hours:** The course will have 3 hours of lectures and 2 hours of exercises (in the computer classroom) per week. The last three weeks of the semester will be devoted to presentations of seminar assignments.

**Language:** Slovenian

## Iterative numerical methods in linear algebra

**Bor Plestenjak**

**Outline:** Many applications lead to problems involving large and often sparse matrices, for which standard methods of numerical linear algebra, such as LU factorization for solving linear systems and QR iteration for computing eigenvalues, become too slow or require too much memory. We will study modern approaches that enable the numerical solution of large systems of linear equations and the computation of eigenvalues and eigenvectors of large matrices.

The two main topics of the course are iterative methods for solving systems of linear equations, where a sequence of approximations converges to the exact solution, and methods for computing selected eigenvalues and eigenvectors of large matrices. We will study Krylov subspace methods, including the well-known conjugate gradient method, preconditioning techniques, and tools from randomized numerical linear algebra. The latter use random projections to transform problems into smaller ones whose solutions, with high probability, differ only slightly from the exact solutions.

The methods for large and sparse matrices will be developed and tested in Matlab.

**Literature:** On `http://www.fmf.uni-lj.si/~plestenjak/Vaje/INMLA/inmla.htm` you can find the lecture and exercise scripts together with a list of suggested literature.

**Prerequisites:** Students of first cycle Mathematics or Financial mathematics get the necessary prior knowledge in compulsory numerical subjects. For mathematicians, a knowledge of Numerical Linear Algebra is handy (but you can also do without it). Students of Interdisciplinary Study of Computer Science and Mathematics who did not choose Numerical methods 2, will have to read something about numerical computation of eigenvalues.

**Assessment:** homeworks (30%), written exam (40%) and oral exam (30%)

**Semester:** summer

**Weekly hours:** 2/1/2

**Language:** Slovenian or English (depending on students enrolled in the course)

## Numerical integration and ordinary differential equations

**Jan Grošelj**

**Outline:** The course covers material that enhances knowledge of mathematics in the field of numerical integration and ordinary differential equations in a practical direction. It introduces students to numerical methods, their analysis and implementation, and familiarizes them with practical problems where specific approaches are particularly effective. This provides strong support for solving various problems in engineering, financial and social sciences, and other fields.

The following topics will be covered: numerical differentiation, Newton-Cotes formulae and their extensions, Gaussian rules, integration in multiple variables, Monte Carlo methods, solving ordinary differential equations, initial value problems, single-step and multi-step methods, stiff differential equations, boundary value problems, and the finite difference method.

**Literature:**

- Jernej Kozak: *Numerična analiza*, DMFA-založništvo, Ljubljana, 2008.
- Bor Plestenjak: *Razširjen uvod v numerične metode*, DMFA-založništvo, Ljubljana, 2015.
- Kendall E. Atkinson: *An introduction to numerical analysis*, John Wiley & Sons, New York, 1978.

**Prerequisites:** Knowledge of the subject matter from the course *Numerical Approximation and Interpolation* is beneficial.

**Assessment:** There are two homework assignments, a written exam, and an oral exam. The homework assignments are assessed in the form of quizzes on a computer and are considered in the grading of the written exam. After passing the written exam, the oral exam must also be completed.

**Semester:** Winter

**Weekly hours:** Lectures and exercises in the format 3/2.

**Language:** Slovenian

## Actuarial mathematics – Life insurance

**Mario Marino**[1]

**Contents**:

(1) *Introduction to life insurance*: biometric risks, financial risks and insurance products.

(2) *Modeling the lifetime*: survival models, mortality laws, life tables, mortality heterogeneity and stochastic mortality dynamics.

(3) *Valuation of insurance benefits*: random present values and actuarial values for different life insurance products and life annuities.

(4) *Pricing life insurance contracts*: the equivalence principle and premium calculation, single premiums, periodic premiums, and expense loadings

(5) *Reserving and policy value*: the actuarial accumulation process and the policy reserve, recursive equations, risk premium and savings premium, expected profit analysis and introductory principles of profit testing.

(6) *Multi-state models and state-dependent benefits*: the Markov model and the evolution of a risk: states, transitions, and cash-flows; increment-decrement tables; distributions of random present values and actuarial values: premiums, reserves, expected profits.

For topics (2) - (6) both time-continuous and discrete time modeling approacches will be considered.

**Literature:**

- Olivieri, A., Pitacco, E. (2015), *Introduction to Insurance Mathematics. Technical and Financial Features of Risk Transfers*, 2nd Edition, EAA Series Springer. (chapters 3-7)
- Dickson, D. C. M., Hardy, M. R., Waters, H. R. (2019). *Actuarial Mathematics for Life Contingent Risks* (3rd ed.). Cambridge: Cambridge University Press. (chapters 1-10, 12-13)

**Prerequisites:** Students are required to have successfully completed a course in probability theory and statistics.

**Assessment:** Written examination, computer lab homework for the seminar part.

**Semester:** Spring

**Weekly hours:** There will be 30 hours of lectures and 30 hours of tutorials. The seminar part of the course (15 teaching hours) will be conducted in collaboration with the Slovenian Actuarial Society.

**Language:** English

[1]Assistant Professor, 1Department of Economics, Business, Mathematics and Statistics "Bruno de Finetti", University of Trieste, Italy

## Financial mathematics 2

**Mihael Perman**

**Outline:** The course is mainly devoted to mathematical tools necessary for derivative valuation in continuous time. Once the tools are available, the course presents the main ideas with examples of real life options and their pricing.

Contents:

1. Overview of prerequisites from analysis and probability.
1.1 Functions of bounded variation.
1.2 Lebesgue-Stieltjes integral.
1.3 Convergence in $L^2$ spaces.
1.1 Maximal inequalities for discrete martingales.
2. Brownian motion.
2.1 Motivation and definition.
2.2 Markov and strong Markov property, reflection principle.
2.2 Brownian martingales.
2.3 Martingales in continuous time, quadratic variation.
2.4 Optional sampling theorem in continuous time.
3. Itô integral.
3.1 Construction, Itô isometry, properties.
3.2 Itô's lemma.
3.2 Localization and local martingales.
3.2 Stochastic integrals with respect to semimartingales.
3.2 General Itô formula.
4. Derivative pricing.
4.1 Self-financing strategies, hedging.
4.2 Black-Sholes model.
4.3 Change of measure, Girsanov theorem.
4.3 Examples of price calculations.

**Literature:**

D. Lamberton, B. Lepeyre, Introduction to Stochastic Calculus Applied to Finance, Chapman & Hall, 2000.
S. E. Shreve, Stochastic Calculus for Finance II, Continuous-Time Models, Springer, 2004.
T. Björk, Arbitrage Theory in Continuous Time, 3rd Edition, Oxford, 2009.

**Prerequisites:** Analysis: partial derivatives, parameter dependent integrals. Probability: independence, expected value, standard distributions, conditional expectation, discrete martingales. Measure theory: Lebesgue integral, monotone and dominated convergence theorem, Fubini theorem, $L^p$ spaces. Financial mathematics: binomial pricing model, definition of derivative securities, one price principle, equivalent measures, completness of models.

**Assessment: 50% written exam, 50% seminar assignment.**

**Semester: winter**

**Weekly hours: 3/2**

**Language: Slovenian, English if there are Erasmus students.**

## Financial mathematics 3 (Interest rate theory)

**József Gáll**[2]

**The aim of the course:** The aim of the course is to discuss modern interest rate models, bond market structures and interest rate related financial assets, such as interest rate and bond derivatives. We discuss several fundamental and widely used interest rate models of the literature: on the one hand both in discrete and in continuous time settings, on the other hand we study short rate and forward interest rate models. Our aim is to show some important results of financial mathematics on this area (e.g. no-arbitrage conditions, asset pricing, fundamental properties and comparison of the models) and also to study specific statistical, practical, economic and financial questions in the models at issue, which may occur in applications.

**Content:** Basic notions, interest rates, yield curves, bond structures, interest rate derivative contracts. Discrete time interest rate models. The Ho-Lee model, binary models introduced by short rate dynamics. Discrete time forward interest rate models, the Heath-Jarrow-Morton (HJM) framework. No-arbitrage conditions, derivative pricing. Continuous interest rate models, arbitrage free family of bond prices, classification of models. Change of numeraire, no-arbitrage in continuous time, forward measures, related stochastic calculus. Some elementary short rate models (Merton, Vasicek, Cox-Ingersoll-Ross, Hull-White), noarbitrage in short rate models, affine term structures. HJM type models in continuous time settings, no-arbitrage criteria and drift conditions. Some special topics: forward rate models driven by random fields, expectation hypotheses, defaultable bonds, pricing problems of certain interest rate derivatives, statistical and fitting questions, calibration and numerical issues. We shall show some examples and scripts written in R in the discussed models, such as examples for the bond market structures under different parametrisations, price and interest rate process evolvements. Furthermore, we discuss some fitting and estimation problems for practice. For the discussion of the fundamental models and theorems we shall mainly use classical monographs such as [1], [2], [3] and some further handouts. On the other hand, we shall also discuss some particular problems in special models for which selected (celebrated) papers of the literature shall be used.

**Literature:**


- Cairns, A.J.G. (2004), Interest Rate Models, Princeton University Press, Princeton and Oxford.
- Brigo, D. and Mercurio, F. (2006), Interest Rate Models – Theory and Practice: With Smile, Inflation and Credit, Springer, Berlin Heidelberg New York.
- Musiela, M. and Rutkowski, M. (2005), Martingale Methods in Financial Modelling, Edition 2, Springer-Verlag, Berlin, Heidelberg.


**Prerequisites:** It is required that you passed a course (or courses) in probability theory, statistics and stochastic processes, and recommended that you passed an introductory course in financial mathematics (fundamental theorems of asset pricing, basic notions and models of option pricing).

**Assessment:** Written examination.

**Semester:** Spring 2027

**Weekly hours:** 2/1/2

In total, there will 30 hours of lectures, 30 hours of tutorial and 15 hours of seminar

**Language:** English

[2]Associate professor, Department of Applied Mathematics and Probability Theory, Faculty of Informatics, University of Debrecen, Hungary, jozsef.gall@inf.unideb.hu

## Financial mathematics 2

**Mihael Perman**

**Outline:** The course is mainly devoted to mathematical tools necessary for derivative valuation in continuous time. Once the tools are available, the course presents the main ideas with examples of real life options and their pricing.

Contents:

1. Overview of prerequisites from analysis and probability.
1.1 Functions of bounded variation.
1.2 Lebesgue-Stieltjes integral.
1.3 Convergence in $L^2$ spaces.
1.1 Maximal inequalities for discrete martingales.
2. Brownian motion.
2.1 Motivation and definition.
2.2 Markov and strong Markov property, reflection principle.
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2.4 Optional sampling theorem in continuous time.
3. Itô integral.
3.1 Construction, Itô isometry, properties.
3.2 Itô's lemma.
3.2 Localization and local martingales.
3.2 Stochastic integrals with respect to semimartingales.
3.2 General Itô formula.
4. Derivative pricing.
4.1 Self-financing strategies, hedging.
4.2 Black-Sholes model.
4.3 Change of measure, Girsanov theorem.
4.3 Examples of price calculations.

**Literature:**

D. Lamberton, B. Lepeyre, Introduction to Stochastic Calculus Applied to Finance, Chapman & Hall, 2000.
S. E. Shreve, Stochastic Calculus for Finance II, Continuous-Time Models, Springer, 2004.
T. Björk, Arbitrage Theory in Continuous Time, 3rd Edition, Oxford, 2009.

**Prerequisites:** Analysis: partial derivatives, parameter dependent integrals. Probability: independence, expected value, standard distributions, conditional expectation, discrete martingales. Measure theory: Lebesgue integral, monotone and dominated convergence theorem, Fubini theorem, $L^p$ spaces. Financial mathematics: binomial pricing model, definition of derivative securities, one price principle, equivalent measures, completness of models.

**Assessment: 50% written exam, 50% seminar assignment.**

**Semester: winter**

**Weekly hours: 3/2**

**Language: Slovenian, English if there are Erasmus students.**

## Optimization in finance

**Nacira Agram**[3]

**Outline: 1) Theoretical part:**

a) Stochastic differential equations and backward stochastic differential equations. Existence and uniqueness. Application to finance, including modelling stock prices, hedging strategies, and dynamic risk measures.

b) Stochastic optimal control, dynamic programming (HJB equation) and Pontryagin's stochastic maximum principle. Examples in finance such as consumption and portfolio optimization are studied.

**2) Numerical part:**

a) Numerical solutions for stochastic differential equations and backward stochastic differential equations. Euler-Maruyama scheme and simulation of stochastic processes. Conditional expectation estimation.

b) Solving optimal stochastic control problems using deep supervised learning algorithms using Pytorch library in Python.

**Literature:**


- Bernt Øksendal. Stochastic differential equations. Springer, 2013.
- Laurent Mazliak. An introduction to probabilistic methods in stochastic control. Lectures at Pescara University, April 1996.
- Kloeden, P. E., & Platen, E. Numerical Solution of Stochastic Differential Equations. Springer. 1992
- Murphy, K. P. Machine Learning: A Probabilistic Perspective. MIT Press. 2012


**Prerequisites:** A listener should be familiar with stochastic processes in continuous time and stochastic integration. A basic programming proficiency is also assumed. Some familiarity with machine learning is desirable but not required.
**Required courses:** Finančna matematika 2, Uvod v programiranje.

**Assessment:** Written examination, homeworks.

**Semester:** Spring

**Weekly hours:** 3/2
Professor Agram will give 30 hours of lectures and 12 hours of seminar. Assistant Jan Rems will conduct 30 hours of tutorials. Three teaching hours of the seminar will be conducted in collaboration with industry.

**Language:** English

[3]KTH University, Stockholm, Sweden, e-mail: nacira@kth.se

# PROBABILITY 2

**Martin Raič**

**Outline:**

**Discrete time Markov chains:** Discrete time random processes – existence and uniqueness. Markov property and transition probabilities. Stopping times, typical examples and counterexamples thereof. Strong Markov property. Markov chains with arbitrary initial state. Induced distributions, stationary distribution. Induced measures, stationary measure. Reversibility – detailed balance condition. Communication between states. Transience and recurrence, positive and null recurrence. Fundamental decomposition of state space. Asymptotic frequency of states. Characterization and existence of stationary distributions; construction of stationary measures. Periodicity of states. Limit theorems: ergodicity, strong law of large numbers. Specific results for the case of finite number of states.

**Continuous time Markov chains:** Regularity conventions. Continuous time Markov property and transition probabilities. Chapman–Kolmogorov equations, semigroup property. Stopping times, strong Markov property. Holding time, jump rate, jump transition probabilities. Embedded jump chain. Infinitesimal behavior, chain generator. Birth processes and explosion phenomenon. Existence and uniqueness, simulation of continuous time Markov chains. Communication between states. Transience and recurrence, positive and null recurrence. Stationary distribution and measure. Reversibility. Limit theorems: ergodicity, strong law of large numbers.

**Applications of Markov chains:** Waiting queue systems (birth & death system, $M/M/1$, introduction into the general theory, some important cases of waiting queue systems). Monte Carlo Markov chains (Bayesian statistics and Monte Carlo simulations, Gibbs sampler and Metropolis–Hastings algorithm, convergence of MCMC algorithms, applications in financial mathematics).

**Literature:**


- J. R. Norris: *Markov Chains.* Cambridge University Press, 1999.
- R. Durrett: *Essentials of Stochastic Processes.* Springer, New York, 2004.
- S. I. Resnick: *Adventures in Stochastic Processes.* Birkhäuser, 1992.
- L. C. G. Rogers, D. Williams: *Diffusions, Markov processes and Martingales. Vol. 1: Foundations.* 2nd ed., Cambridge University Press, 2000.
- G. Grimmett, D. Stirzaker: *Probability and Random Processes.* 3rd ed. Oxford University Press, 2001, 2009.


**Prerequisites:** basic probability theory, basic ordinary differential equations.

**Assessment:** Type: written exam or 2 midterm type exams, oral exam which can be partially replaced by theoretical tests. Grading: 1–5 (fail), 6–10 (pass) (according to the Statute of UL).

**Semester:** Winter

**Weekly hours:** 3/2

**Language:** Slovenian or English (depending on students enrolled in the course).

## Statistics 2

**Jaka Smrekar**

**Outline:** Statistics is an extremely important field with a wide spectrum of application in finance, medicine, computer science, artificial intelligence and elsewhere. The goal of the course is a structured overview of some foundational methods of statistical inference with emphasis on application in statistical practice.

(1) **Introduction.** Sufficiency, completeness, exponential families of distributions.
(2) **Estimation.** Optimal unbiased estimation. Estimation in linear models. Asymptotic properties of estimators, asymptotic properties of estimators in linear models.
(3) **The method of maximum likelihood.** Basic definitions and examples. Asymptotic properties. Maximum likelihood estimator in generalized linear models and its aymptotic properties.
(4) **Hypotheses testing.** Uniformly most powerful tests and the Neyman-Pearson approach. Testing in exponential and normal models. Testing in normal linear models. Likehood ratio tests, testing for homogeneity and independence. Testing in nonparametric models, rank-based tests.
(5) **Confidence sets.** Construction: pivotal quantity, inversion of the non-rejection region. Properties.

**Literature:**

- G.G. Roussas: A course in mathematical statistics, 2nd edition, Academic Press, 1997.

**Prerequisites:** Probability 1 and Statistics 1 or Probability and Statistics.

**Assessment:** Written exam and oral exam.

**Semester:** Winter.

**Weekly hours:** 3/2

**Language:** Slovenian.

## Stochastic processes 2

**Matija Vidmar**

**Outline:** We aim first to study the theory of (sub-/super-)martingales: filtrations and stopping (optional) times; optional sampling and stopping; maximum and upcrossing inequalities; convergence theorems; regularization of sample paths. The second main ingredient of the course are the fundamentals of the so-called general theory of stochastic processes: progressive, predictable and optional sigma-fields; classification of stopping times; début, section and projection theorems; the Doob-Meyer decomposition. Lastly, time-permitting, we look at key elements of the theory of stochastic integration: quadratic variation of continuous local martingales, integration against continuous semimartingales, Itô's formula; integration against general semimartingales, the Bichteler-Dellacherie theorem.

**Literature:**

- Olav Kallenberg: Foundations of modern probability, Springer Cham, 2021 [relevant parts thereof].

**Prerequisites:** Basic measure-theoretic probability or at least measure theory, including conditional expectation (the content of the arXiv e-print arXiv:2604.01685 suffices). Knowledge of the more basic notions/results of the (undergradate-level) calculus of probability (e.g. Borel-Cantelli lemmas, independence, characteristic functions etc.).

**Assessment:** Written exam for the exercise classes; written or oral exam for the lectures (to be determined).

**Semester:** Second.

**Weekly hours:** 3/2.

**Language:** Slovenian or English, subject to agreement (automatically English if attended by students that will have enrolled in the course but do not know Slovene).

## Advanced Machine Learning

**Ljupčo Todorovski**

**Outline:** The aim of the course is to introduce students to advanced machine learning (ML) methods: on the one hand, understanding their structure and functioning, and on the other, their usefulness for solving practical problems. The course covers a selection of three to four thematic modules. The selection is adapted to the current and most relevant results within each module. The following is an indicative selection of four modules for the next academic year:

(1) **Deep ML**: advanced architectures of neural networks (NNs), from recurrent NNs to attention mechanisms and transformers, graph and recursive NNs for structured data, and methods for improving NN training.
(2) **Generative Models**: the ability to generate new data instances similar to those in the training set, paradigms of generative modelling, including variational autoencoders (VAEs) and generative adversarial networks (GANs).
(3) **Equation Discovery or Symbolic Regression**: algorithms for the automatic discovery (machine learning) of the structure and parameters of mathematical models from data, paradigms of evolutionary optimization, probabilistic context-free grammars, generative models, and stochastic search.
(4) **Meta-learning and Automated ML**: machine learning based on the results of machine learning, which helps select optimal algorithms and hyperparameters for solving new problems; paradigms of meta-learning, automated machine learning, and hyperparameter optimization.

**Literature:**


- Prince, S. J. D. (2023) *Understanding Deep Learning*. MIT Press. Available at `https://udlbook.github.io/udlbook/`
- Tomczak, J. M. (2022) *Deep generative modeling*. Springer.
- Mežnar, S., Džeroski, S. in Todorovski, L. (2023) Efficient generator of mathematical expressions for symbolic regression. *Machine Learning, 112(10)*: 4563–4596. DOI: 10.1007/s10994-023-06400-2
- He, X., Zhao, K., Chu, X. (2021) AutoML: A survey of the state-of-the-art. *Knowledge-Based Systems, 212*: 106622. DOI: 10.1016/j.knosys.2020.106622. Available at `https://arxiv.org/abs/1908.00709`


**Prerequisites:** Programming in the Python programming language, knowledge of the basic machine learning algorithms, which students acquire in the undergraduate elective course *Selected chapters from data analysis*.

**Assessment:** Homework and oral exam. In the case of many registered students, a written exam might be required.

**Semester:** summer

**Weekly hours:** 2/1/2

**Language:** Slovenian

## Teorija izračunljivosti (Computability theory)

**Alex Simpson**

**Outline:** This course addresses the fundamental question: what kinds of problem are *computable*, that is, roughly speaking, are amenable to solution by a computer? Our starting point will be diverse mathematical definitions of *computability* given by several people including Alan Turing and Kurt Gödel in the 1930s. A key result is that all definitions are equivalent. This motivates the *Church-Turing thesis*: there is one absolute notion of computability. One can thus classify mathematical problems according to their computability status. For example, the *decidable* problems are those for which an answer can be computed, whereas the *undecidable* problems are those whose answer cannot be found by an algorithm. Turing's notorious *halting problem* is an archetypal problem in the latter class. Many other famous mathematical problems reside in this class as well.

Traditional computability theory addresses computational problems involving *discrete data*. The course will also study the extension of computability to *continuous data*, such as functions, real numbers, etc. This allows computability questions to be directed at continuous mathematics, and also, more practically, corresponds to a form of computation with continuous data (e.g., real numbers) that avoids rounding errors.

(1) Turing machines, the universal Turing machine, computability and undecidability.
(2) Other models of discrete computation and the Church-Turing thesis.
(3) Classifying sets and functions according to their computability properties.
(4) Type 2 Turing machines for computation with continuous data.
(5) Computable functions on real numbers and other continuous data types.

**Literature:**


- N.J. Cutland. Computability: An Introduction to Recursive Function Theory. Cambridge University Press. 1980.
- M. Sipser. Introduction to the Theory of Computation. Cengage Learning. 3rd Edition, 2012.
- A.M. Turing. On Computable Numbers, with an Application to the Entscheidungsproblem. Proceedings of the London Mathematical Society 2 42:230–65, 1937.
- K. Weihrauch. Computable Analysis: An Introduction. Springer-Verlag, Berlin/Heidelberg, 2000.


**Prerequisites:** The subject *Logika in Množice* (logic and sets) from the undergraduate (1st level) mathematics programme, or equivalent knowledge.

**Assessment:** Written and oral exams.

**Semester:** Winter.

**Weekly hours:** 3/2

**Language:** Slovenian or English.

## Computational Geometry

**Sergio Cabello**

**Outline:** The course covers basic algorithms in Computational Geometry, an area dedicated to algorithmic problems where the input data has a geometric flavor. In the lectures we will cover the following topics in detail:

- Segment intersection. Sweep algorithms.
- Polygons and triangulations of polygons.
- Convex sets. Algorithms to compute the convex hull of points in the plane.
- DCEL. Point location.
- Voronoi diagrams. Fortune's algorithm.
- Delaunay triangulation in the plane. Incremental construction. Interpretation in 3 dimensions.
- Data structures for points.
- Duality and line arrangements.

In the seminar we will discuss four additional topics that are decided later on and may change from year to year.

**Literature:**

- Mark de Berg, Otfried Cheong, Marc van Kreveld, Mark Overmars: Computational geometry: algorithms and applications, Springer 2008.
- There are several good lecture notes and other materials available.

**Prerequisites:** Basic knowledge of algorithms and data structures.

**Assessment:**

- Students have to participate in the seminar, pass a written exam of exercises and pass an oral exam.
- Knowledge about the topics discussed in the seminar is checked also at the oral exam. Each student can reduce the content of the seminar to two topics of their choice.
- The grade takes into account the written exam of exercises ($\sim$40%), the seminar ($\sim$20%) and the oral exam ($\sim$40%).

**Semester:** winter

**Weekly hours:** 2/1/2 in average. The seminar takes place in blocks of three hours.

**Language:** English. Oral exams can be in Slovenian or English.

## Computational methods for group theory doc. dr. Micael Alexi Toledo Roy

**Outline:** This course introduces the theory of groups from first principles while developing practical computational skills using the computer algebra system GAP. Students will learn the fundamental concepts and methods of group theory alongside their algorithmic implementation to solve problems in algebra. The theoretical and practical aspects of the course will take place in parallel: we will learn how to implement concepts of algebra in GAP as we learn them.

**Literature:**

- An Introduction to the Theory of Groups, Joseph J. Rotman, Springer, 1995.
- GAP - Reference Manual.

**Prerequisites:** Basic knowledge of algebra and programming

**Assessment:** Theoretical exam and practical homework assignments.

**Semester:** Winter

**Weekly hours:** 3/2

**Language:** English

## Logika (logic)

**Alex Simpson**

**Outline:** The course studies first-order logic, its proof theory and model theory, leading to Gödel's celebrated incompleteness theorems.

This material touches on matters of fundamental significance in mathematics which have bearing on the nature of mathematical reasoning and of mathematical truth, and on the need for creativity in mathematics. Gödel's *completeness theorem* demonstrates that we can completely describe all legitimate rules for reasoning from a set of axioms. His *incompleteness theorems*, in contrast, show that it is impossible to provide a consistent set of axioms capturing all truths of mathematics. And the Church/Turing *undecidability theorem* shows that it is impossible to write a computer program that will automatically "decide" whether a mathematical statement is true or false.

(1) Syntax and semantics of first-order logic.
(2) Examples of first-order theories and quantifier elimination.
(3) Proof systems and Gödel's completeness theorem.
(4) Compactness, the Löwenheim-Skolem theorem, basic model theory.
(5) Arithmetic and the Church/Turing undecidability theorems.
(6) Gödel's incompleteness theorems.

**Literature:**

- N. Prijatelj: Osnove matematične logike, 2. del: Formalizacija, DMFA Slovenije, Ljubljana, 1992.
- N. Prijatelj: Osnove matematične logike, 3. del: Aplikacija, DMFA Slovenije, Ljubljana, 1994.
- W. Rautenberg: A Concise Introduction to Mathematical Logic. Universitext, 2009.

**Prerequisites:** The subject *Logika in Množice* (logic and sets) from the undergraduate (1st level) mathematics programme, or equivalent knowledge.

**Assessment:** Written and oral exams.

**Semester:** Summer.

**Weekly hours:** 3/2

**Language:** Slovenian or English

## Selected Topics in Optimisation

Lecturer: Nino Bašić

**Outline:** Combinatorial optimisation problems arise throughout mathematics and computer science. They may be approached using general methods of mathematical programming or specialised algorithms that exploit problem structure. The course covers selected methods of discrete optimisation and applications to covering problems, cellular automata, and graph drawing, with an emphasis on mathematical modelling and comparison of approaches. Topics include:

(1) **Integer programming**: binary and mixed-integer formulations, linear relaxations, integrality gaps, formulation strength, and symmetry breaking.
(2) **Valid inequalities and polyhedral methods**: the integer hull, cover inequalities, and Gomory and Chvátal–Gomory cuts.
(3) **Packing, covering, and partitioning problems**: *set packing*, *set covering*, and *set partitioning* formulations, exact cover, and tilings of regions.
(4) **Algorithm X and Dancing Links**: Knuth's exact-cover algorithm and its comparison with integer programming and other general methods.
(5) **Inverse problems in cellular automata**: modelling Conway's Game of Life, finding predecessors and periodic patterns, and comparing integer-programming, logical, and specialised formulations.
(6) **Optimisation methods for graph drawing**: drawing quality criteria, energy and stress models, spring-based and force-directed methods, and the Fruchterman–Reingold and Kamada–Kawai algorithms.
(7) **Planarity and planar embeddings**: combinatorial embeddings, rotation systems and faces, planarity testing and embedding construction, certificates of planarity and non-planarity, the Fáry–Wagner theorem, and Tutte's barycentric method.
(8) **Computational implementation and comparison**: software tools for mathematical optimisation, implementation of selected algorithms, and experimental comparison of general optimisation and specialised combinatorial methods.

The depth of treatment will reflect student interests: some topics will be studied in detail, others mainly conceptually and through selected examples.

**Literature:**

- L. A. Wolsey: *Integer Programming*, Wiley, 2020.
- B. Korte, J. Vygen: *Combinatorial Optimization*, Springer, 2018.
- R. Tamassia: *Handbook of Graph Drawing and Visualization*, Chapman & Hall/CRC, 2014.

**Prerequisites:** Basic knowledge of linear algebra, graph theory, algorithms and data structures, and programming. Undergraduate-level knowledge of linear programming (Optimization Methods or Optimization 1) is recommended.

**Assessment:** Homework assignments (including one project) and a written examination (or two midterm tests).

**Semester:** Spring

**Weekly hours:** 3/2

**Language:** Slovenian

## Symbolic Computation

**Daniel Smertnig**

**Outline:**

Symbolic computation (also known as computer algebra) is concerned with the exact, algorithmic representation and manipulation of mathematical objects such as polynomials, rational functions, and formal expressions, in contrast to numerical methods that work with floating-point approximations. It provides the underlying algorithms for computer algebra systems such as Oscar, SageMath, GAP, PARI/GP, and others.

In particular, we plan to cover the following topics:

- rewrite systems (reduction relations, Newman's lemma, completion);
- operations with polynomials (modular arithmetic, Hensel lifting, polynomial factorization and decomposition);
- operations with polynomial ideals (monomial orders, Gröbner bases, solving systems of algebraic equations);
- solving linear difference and differential equations (polynomial solutions, hypergeometric and hyperexponential solutions).

Time permitting, we may also cover some additional key algorithms, such as the LLL algorithm for lattice basis reduction.

**Literature:**

- Cox, D., Little, J., O'Shea, D. *Ideals, varieties, and algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra*, 5th ed., Springer, 2025.
- von zur Gathen, J., Gerhard, J. *Modern Computer Algebra*, 3rd ed., Cambridge University Press, 2013.
- Kauers, M., Paule, P. *The concrete tetrahedron: Symbolic sums, recurrence equations, generating functions, asymptotic estimates.*, Springer, 2011.

**Prerequisites:** Acquaintance with basic structures from abstract algebras (in particular, polynomial rings and ideals).

**Assessment:** Oral (theory) and practical homework/project work.

**Semester:** Spring

**Weekly hours:** 2/1/2

**Language:** English, possibly with some Slovenian elements.

## Modern physics

**izr. prof. dr. Sašo Grozdanov**

**Outline:** 'Modern physics' deals with the main theoretical concepts and related mathematical structures that were discovered in the physics of the 20th century. The course is divided into three parts, in which we will discuss the following topics:

The electromagnetic field:

- Electric and magnetic fields;
- The Maxwell equations;
- Electromagnetic waves;
- Magnetic monopoles;

The theory of relativity:

- The special theory of relativity: spacetime, the Lorentz transformations, the Minkowski metric and the Poincaré group;
- Electromagnetism and the theory of relativity: the Maxwell equations in covariant form;
- The Maxwell equations in the language of differential forms;
- Introduction to classical field theory (the Lagrangian and the action);
- Electromagnetism as gauge field theory;
- Curved spacetime, geodesics and the introduction to the general theory of relativity;
- What are black holes?;

Quantum phyiscs:

- Why do we need quantum physics?;
- Wave-like properties of particles, the Schrödinger equation and the probabilistic interpretation;
- The postulates of quantum mechanics and the Heisenberg relations;
- Examples of quantum systems: the harmonic oscillator and the Hydrogen atom;
- Relativistic quantum mechanics, the Dirac equation, spin and anti-matter;
- Introduction to the Feynman path integral: why is classical physics the limit of quantum physics?
- What is quantum field theory and what is the standard model of particle physics?

**Literature:** Lecture notes will be published online. A variety of optional books and notes will be recommended for different topics during lectures.

**Prerequisites:** We expect the students to have the knowledge of physics at the level of related subjects taught during the 1st (Bachelors) level of the undergraduate mathematics studies.

**Assessment:** Your understanding of the course materials will be assessed with mandatory homework assignments (in total, approximately 5-6 homework assignments). The written exam can be written in class or replaced by an additional homework assignment (depending on the agreement with the students). At the end, the final assessment will also include an oral exam.

**Semester:** Winter

**Weekly hours:** 3/2

**Language:** English (or Slovenian; depending on the agreement with the students)

# Bridge

**Extracurricular activity, 3 ECTS**

**Barbara Drinovec Drnovšek**

**Outline:** Bridge is a card game for 4 players, two against two as partners, played with the usual deck of 52 cards. It consists of two parts: bidding and card play. When bidding, we try to predict as accurately as possible how many tricks my partner and I will take and which color will be the trump siut. We only see our cards; we communicate with the partner through bidding. In the second part of the game, the winning pair tries to win at least as many tricks as they bid. The opposing pair tries to prevent this.

The game develops logical thinking, the ability to make quick decisions, social skills, and partnership.

In the course, we will learn the basic rules of minibridge and bridge. We will learn the basics of bidding. We will use the information from the auction in the card play. We will learn the fundamental elements of card play technique: card combinations, blocking and unblocking, maintaining communication, and preventing communication between opponents. We will focus on attacking and using the permitted methods of communication between partners in the defence.

We will play one tournament in minibridge and two in bridge.

**Prerequisites:** Enrollment in the Bachelor or Master study program of the University of Ljubljana. No prior knowledge is required.

**Assessment:** Student is evaluated with "passed"/"failed" grades. 75% attendance in the course is needed for taking the exam. With two successfully played tournaments, the student passes the exam.

**Semester:** winter, on Wednesdays at 16:15.

**Weekly hours:** 1/2
One hour of lectures will be followed by two hours of exercises classes. Students will learn to apply the acquired theoretical knowledge in practice.

**Language:** Slovenian

**Other** A student can choose an extracurricular activity within the framework of general elective courses. Enrollment in extracurricular activities will take place from the beginning of enrollment at the FMF until the vacancies are filled.

## Voluntary work in teaching support

**Damjan Kobal**

**Outline:** Young people from socially disadvantaged backgrounds are often less successful in their academic work and need learning support, which they cannot afford. On the other hand, for such children, academic success can be motivation and the only hope for getting out of negative social-family cycles.

In cooperation with relevant humanitarian organisations, such as *Zveza prijateljev mladine Ljubljana Moste-Polje*, *Mladinski dom Malči Beličeve* or *Slovenska filantropija*, the student carries out approximately 30 hours of individual or group instruction or other accompanying work for young people within the framework of the organised activities of the relevant humanitarian organisations. The student keeps a simple diary of the content and scope of the work and submits a simple report at the end.

**Prerequisites:** All students enrolled in the first or the second cycle of any study programme of the University of Ljubljana have sufficient prior academic knowledge for the expected work.

**Assessment:** A pass/fail grade is awarded on the base of relevant student's engagement.

**Semester:** Winter or Spring

**Weekly hours:** As agreed

**Language:** Introductory information in Slovenian or English. Practical learning assistance is expected to be mostly in Slovenian. Students who do not speak Slovenian, might be able to help in more general social activities of the relevant humanitarian organizations.

## Kategorična logika (Categorical Logic))

**Alex Simpson**

**Outline:** The course will study category-theoretic models of logics of interest in mathematics and computer science. The course will be self-contained.

(1) Cartesian-closed categories and typed $\lambda$-calculus.
(2) Example: functor categories.
(3) Example: categories of assemblies.
(4) Strong monads and calculi for computational effects.
(5) Example: categories of algebras.
(6) Locally cartesian-closed categories and dependent type theory.
(7) Regular categories and the logic of subobjects.
(8) Quasitoposes and toposes.
(9) Example: categories of sheaves.
(10) Example: realizability toposes.
(11) Fibrations and hyperdoctrines.

**Literature:**

- S. Mac Lane and I. Moerdijk. Sheaves in Geometry and Logic. Springer-Verlag, 1992.
- B. Jacobs. Categorical Logic and Type Theory. North Holland, 1999
- S. Awodey. Categorical Logic. Lecture notes, 2024.

**Prerequisites:** General mathematical maturity, as befitting a PhD student.

**Assessment:** Seminar participation and oral exams.

**Semester:** Winter.

**Weekly hours:** 2/1 (lectures: 2, seminar: 1)

**Language:** English

## Cohomological methods in complex analysis

**Petar Pavešić, Marko Slapar**

**Outline:** The course is an introduction to cohomological methods in complex analysis. We will first discuss some basic cohomological techniques, including optional topics depending on the background and interests of the students. Second part of the course will present several fundamental results in complex analysis that are derived using cohomological methods.

(1) Good covers, existence, cofinality, properties (optional: nerve theorem). Cohomology theory associated to a cover, properties. Direct limits of groups, exactness. Čech cohomology, relation with DeRham cohomology, in particular between the cup-product in Čech cohomology and the wedge product of differential forms. Optional: basics on spectral sequences, spectral sequence of a double complex.

(2) Analytic sheaves and Oka's syzygy theorem, Heftungslemma of Cartan and amalgamation of analytic sheaves, coherent analytic sheaves Stein manifolds and Cartan's Theorems A and B, Dolbeault cohomology, Cousin problems and applications.

**Literature:**


- R. Bott, L. Tu, Differential forms in Algebraic Topology, GTM 82 Springer, (1982).
- H. Grauert and R. Remmert, Theory of Stein spaces. Transl. from the German by Alan Huckleberry. Reprint of the 1979 ed. Berlin: Springer (2004).
- R. C. Gunning and H. Rossi: Analytic functions of several complex variables. Reprint of the 1965 original. Providence, RI: AMS Chelsea Publishing (2009).


**Prerequisites:** Some of the topics will require methods that are usually treated in standard courses in algebraic topology and complex analysis. Whenever necessary, these topics will be briefly recalled during the course.

**Assessment:** Seminar and oral exam.

**Semester:** summer

**Weekly hours:** 2/0/1

**Language:** Slovenian or English (depending on students enrolled in the course)

## Extension Theory of Symmetric Operators with Applications

**Aleksey Kostenko**

**Outline:**

The main focus is on the extension theory of symmetric operators in Hilbert spaces with applications to linear differential operators (Hamiltonians with point interactions, quantum graphs and elliptic PDEs). The first part of the course will be devoted to the classical approach developed by J. von Neumann, K. Friedrichs and M. G. Krein. Then we shall proceed with a new powerful approach to the self-adjoint extension theory based on the notion of a boundary triplet for the adjoint of a symmetric operator. Pioneered by J.W. Calkin, M. Vishik and M. Birman, this approach turned out to be especially useful for differential operators.

**Literature:**


- M. Sh. Birman and M. Z. Solomyak, *Spectral Theory of Self-Adjoint Operators in Hilbert spaces*, D. Reidel Publ., Kluwer, 1987.
- V. Derkach and M. Malamud, *Extension Theory of Symmetric Operators and Boundary Value Problems*, Springer, 2025.
- K. Schmüdgen, *Unbounded Self–Adjoint Operators on Hilbert Space*, Springer, 2012.
- B. Simon, *A Comprehensive Course in Analysis, Part 4: Operator Theory*, Amer. Math. Soc., Providence, RI, 2015.


**Prerequisites:** Basics of functional analysis. Basics of ODEs and PDEs are desirable but not necessary.

**Assessment:** Written exam.

**Semester:** Winter

**Weekly hours:** 2

**Language:** English

## Structure and randomness in graph theory and additive combinatorics

**Urban Jezernik**

**Outline:** This PhD course gives a modern introduction to a central theme in contemporary combinatorics: the dichotomy between structure and pseudorandomness, developed in parallel in graph theory and additive combinatorics, and linked by a shared toolkit of methods and viewpoints. We will closely follow Yufei Zhao's new textbook Graph Theory and Additive Combinatorics: Exploring Structure and Randomness, aiming to cover most of its material.

On the graph theory side, we develop foundational techniques from extremal and probabilistic combinatorics, starting from classical forbidden-subgraph problems (e.g. Turán-type questions) and moving to structural tools such as Szemerédi's regularity method, counting/removal ideas, and models of randomness via pseudorandom (quasirandom) graphs and spectral methods. We then study analytic frameworks that organize large-scale graph structure, including graph limits/graphons and graph density homomorphism inequalities, emphasizing how analytic inequalities encode combinatorial constraints.

On the additive combinatorics side, we focus on landmark results and the ideas behind them, especially Roth-type theorems and 3-term arithmetic progressions, Fourier-analytic methods in finite abelian groups and the integers, and the broader "regularity" philosophy in arithmetic settings. We then study the structure of sumsets (small doubling, Freiman-type structure) and the sum–product phenomenon, before discussing how dense results transfer to sparse pseudorandom settings, culminating in key concepts surrounding Szemerédi/Green–Tao–type transference. Throughout, we emphasize how the same conceptual template (decomposing objects into structured and pseudorandom parts) appears across proofs and applications, and how graph-theoretic and additive viewpoints reinforce each other.

The course is designed as a gateway to a broad toolkit (combinatorial, spectral, analytic, and Fourier methods) and as preparation for reading current research at the interface of extremal combinatorics and additive number theory.

**Literature:**

- Zhao, Yufei. *Graph theory and additive combinatorics: exploring structure and randomness.* Cambridge University Press, 2023.

**Prerequisites:** /

**Assessment:** Reading homework, oral exam.

**Semester:** Spring

**Weekly hours:** 2/1

**Language:** English