Homotopy principle in analysis and geometry

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Homotopy principle or h-principle is a principle that helps us turn some soft (formal) solutions of a particular problem into an actual solution. The notion of h-principle was introduced and developed by M. Gromov in the 1970s an 1980s. While finding actual solutions of a problem often means finding solutions to some differential equalities or inequalities (conditions on derivatives), finding formal solutions turns out to be of a purely homotopy-theoretic nature. For example, showing that two immersions into an euclidian space are regularly homotopic, one must show the existence of a (smooth) one dimensional family of maps with nondegenerate derivatives connecting the two immersions. A much easier problem of finding a formal solution to the same problem means showing that the two derivatives of the given maps are homotopic in the appropriate space of real matrices with maximal rank, meaning that we must (only) understand the space of homotopy classes of maps into the space of matrices.

In the first part, we will introduce some basic notions of jet bundles of a fibration and define differential relations as subsets of jet bundles. We will show that many classical and perhaps less classical problems in geometry, topology and analysis can be formulated in terms of a differential relation in an appropriate jet bundle. We will then show some general methods of determining whether a differential relation satisfies certain types of h-principle and apply the theory in a variety of examples from differential, CR, symplectic and contact geometry.

Literature

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