Geometric Group Theory

The basic idea of geometric group theory is to prove that a group G has a hypothesized property by making it act in a certain way on a metric space with additional geometric properties. This approach has turned out to be extremely fruitful and has provided the tools to cope with group-theoretical problems that had previously been inaccessible. For example, the completely combinatorial problem of whether or not a Coxeter group

$$\Gamma = \langle s_1, \ldots, s_n \mid (s_i s_j)^{m_{ij}} = 1 \rangle ,$$

(where the $m_{ii} = 1$ and $m_{ij} \geqslant 2$ for $i \neq j$ are natural numbers or ∞ if the particular relation is omitted) has a solvable conjugacy problem, was solved only by showing that it acts properly and cocompactly by isometries on a CAT(0) polyhedral complex.

In the first part of the course, we will consider the basic notions, starting with the concept of a group presentation and Cayley graph associated to it, continuing with group actions, quasi-isometries, and quasi-isometric invariants.

In the second part of the course, we will study group actions on non-positively curved spaces, rounding the theory up with applications in the word problem and the conjugacy problem. Time allowing, we will touch on hyperbolic groups in the sense of Gromov.

Literature

(1) M. R. Bridson, *Non-positive curvature and complexity for finitely presented groups.* International Congress of Mathematicians. Vol. II, 961–987, Eur. Math. Soc., Zürich, 2006.

(2) Clara Löh, *Geometric Group Theory. An Introduction.* Universitext, Springer, 2018.

(3) M. R. Bridson, A. Haefliger, *Metric spaces of non-positive curvature.* Grundlehren der Mathematischen Wissenschaften 319. Springer-Verlag, Berlin, 1999.