Izbrana poglavja iz računalniške matematike: Category theory

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Outline: The course studies the fundamental concepts and constructions of category theory, paying particular attention to how they apply to and illuminate a variety of concepts across different areas of mathematics, including algebra, analysis, computer science, discrete mathematics, geometry, probability theory, quantum theory and topology. Typically, disparate concrete constructions in different areas motivate a single category-theoretic definition that unifies them, and this general description, in turn, leads to the discovery of other mathematical manifestations of the same phenomenon. Important properties of the concrete constructions often follow as instances of general theorems about the corresponding category-theoretic construction. Furthermore, the category-theoretic viewpoint sometimes suggests far reaching generalisations of the original mathematical concepts (for example, the notion of sheaf for a Grothendieck topology).

The course is organised around the list of general category-theoretic notions that will be considered. The development will be motivated and illustrated by numerous examples taken from diverse areas of mathematics.

(1) Categories, functors and natural transformations.
(2) Limits and colimits.
(3) Monoidal categories.
(4) Adjunctions.
(5) Closed categories.
(6) Internal algebraic structures.
(7) Monads and their algebras.
(8) Presheaves and the Yoneda lemma.
(9) Ends, coends and Kan extensions.
(10) The co-Yoneda lemma and its consequences.
(11) Grothendieck topologies and Grothendieck toposes.
(12) Elementary toposes.

Assessment: Homework exercises and an oral exam.

Literature:

- E. Riehl. Category Theory in Context. Dover Modern Math Originals, 2017.
- T. Leinster. Basic Category Theory. Cambridge Studies in Advanced Mathematics, 2014.
- S. Mac Lane. Categories for the Working Mathematician. Springer-Verlag, second edition, 1998.
- S. Mac Lane and I. Moerdijk. Sheaves in Geometry and Logic. Springer-Verlag, 1992.