HOMOLOGY, PERSISTENCE AND MAGNITUDE

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Content: Persistent homology was developed as a method to study the structure of large data sets from a topological perspective. The data sets usually have the structure of a finite metric space and are therefore discrete from a topological perspective. However, based on the metric, we can assign various filtered topological objects to them. Using the functoriality of the invariants of algebraic topology, we can then track the changes in topological features along the filtration and thus extract the desired information.

Magnitude is a numerical invariant of metric spaces which arose in category theory in connection with attemps to define a notion of Euler characteristic of an (enriched) category. Despite this rather abstract motivation, it turns out that the magnitude of a metric space contains geometric information such as volume, surface area, or Minkowski dimension. Magnitude can also be categorified: this means that it can be understood as the Euler characteristic of the so-called magnitude homology associated to the metric space.

In the course, we will study both invariants and the connections between them. We will begin by looking at simplicial complexes and simplicial sets and reviewing the basics of homology. Then we will define persistent homology for various filtrations. We will discuss the general structure of persistence modules. We will define the magnitude of metric spaces and discuss its properties. We will study the specific case of magnitude of graphs and use this to motivate the idea of magnitude homology of graphs and its generalizations. Finally, we will look at the connections between the two concepts: we will study the blurred magnitude homology and persistent magnitude. (This programme is provisional, depending on the time available and the wishes of the attendees.)

Literature: The literature is extensive, the course will mostly be based around the following articles:

- Frederic Chazal, Vin de Silva, Marc Glisse & Steve Oudot. The structure and stability of persistence modules.
- Tom Leinster & Mark Meckes. The magnitude of a metric space: from category theory to geometric measure theory.
- Tom Leinster & Michael Shulman. Magnitude homology of enriched categories and metric spaces.
- Nina Otter. Magnitude meets persistence: homology theories for filtered simplicial sets.
- Dejan Govc & Richard Hepworth. Persistent magnitude.

Prerequisites: General mathematical knowledge obtained during the first cycle of Mathematics is recommended. It is useful (but not strictly necessary) to know some of the basic constructions of Algebraic Topology and Category Theory; however, these will be reviewed during the course. The chapter about magnitude will also require a little bit of knowledge of Analysis.

Delivery/Examination: Lectures. The planned mode of examination is by homework assignments or project with an oral defense if needed.

Semester: summer

Language: Slovene (or English upon agreement with the students)