

Mapping signed sparse graphs to (K_{2k}, M)

Zhouningxin Wang

Abstract. A signed graph (G, σ) is a graph G (loops and multi-edges allowed) together with an assignment $\sigma : E(G) \rightarrow \{+, -\}$. A homomorphism of a signed graph (G, σ) to (H, π) is a mapping of vertices and edges of G respectively to the vertices and edges of H such that the adjacencies, the incidences, and the signs of closed walks are preserved. Motivated by reformulations of the k -coloring problem in this language, and especially in connection with results on 3-coloring of planar graphs, such as Grötzsch's theorem, we consider bounds on the maximum average degree which are sufficient for mapping signed graphs to the signed graph (K_{2k}, M) ($k \geq 3$) where the negative edges form a perfect matching. In this talk, we show that for $k = 3$ the maximum average degree strictly less than $\frac{14}{5}$ is sufficient and for values of $k \geq 4$, we find the best maximum average degree bound to be 3. Moreover, we discuss the applications of our work to signed planar graphs and propose questions similar to Steinberg's conjecture for the class of signed bipartite planar graphs.