

# Edge Metric Dimension and Mixed Metric Dimension of Graphs

Aleksander Kelenc

Faculty of Electrical Engineering and Computer Science  
University of Maribor, Koroška cesta 42, 2000 Maribor, Slovenia.

Institute of Mathematics, Physics and Mechanics  
Jadranska ulica 19, 1000 Ljubljana, Slovenia.

e-mail: *aleksander.kelenc@um.si*

## Abstract

Let  $G = (V(G), E(G))$  be a connected graph, let  $v \in V(G)$  be a vertex and let  $e = uv \in E(G)$  be an edge. A vertex  $w \in V(G)$  distinguishes two edges  $e_1, e_2 \in E$  if  $d_G(w, e_1) \neq d_G(w, e_2)$ . A set  $S$  of vertices in a connected graph  $G$  is an edge metric generator for  $G$  if every two edges of  $G$  are distinguished by some vertex of  $S$ . The smallest cardinality of an edge metric generator for  $G$  is called the edge metric dimension and is denoted by  $dim_e(G)$ . The mixed metric dimension is a graph invariant similar to the edge metric dimension that deals with distinguishing the elements (vertices and edges) of a graph. A vertex  $w \in V(G)$  distinguishes two elements of a graph  $x, y \in E(G) \cup V(G)$  if  $d_G(w, x) \neq d_G(w, y)$ . A set  $S$  of vertices in a connected graph  $G$  is a mixed metric generator of  $G$  if every two elements  $x, y \in E(G) \cup V(G)$  of  $G$  are distinguished by some vertex of  $S$ . The smallest cardinality of a mixed metric generator of  $G$  is called the mixed metric dimension and is denoted by  $dim_m(G)$ . The problems of determining  $dim_e(G)$  and  $dim_m(G)$  are NP-hard in the general case. In this talk, we present the concept of edge metric dimension and its mathematical properties. We make a comparison between the edge metric dimension and the standard metric dimension of graphs. Moreover, we present some bounds and closed formulae for the edge metric dimension of several classes of graphs. We consider the structure of mixed metric generators and characterize graphs for which the mixed metric dimension equals the trivial lower and upper bounds. We also give results on the mixed metric dimension of certain families of graphs and present an upper bound with respect to the girth of a graph. Finally, we present some recent results on edge metric dimension and mixed metric dimension of hypercubes.