ℓ-Facial Edge-Coloring of Plane Graphs

A cyclic coloring of a plane graph is a vertex coloring in which all vertices incident with the same face receive distinct colors. It is conjectured that every plane graph with maximum face size Δ^* admits a cyclic edge-coloring with at most $\lfloor 3\Delta^*/2 \rfloor$ colors. Since, in the case of plane triangulations this coloring corresponds to a proper coloring, the Four Color Theorem implies the conjecture for $\Delta^* = 3$.

As a generalization of cyclic coloring, a k-facial coloring was introduced, where vertices at facial distance at most k receive distinct colors. An open conjecture states that every plane graph admits a k-facial coloring with $3k+1$ colors.

Similarly, an ℓ-facial edge-coloring of a plane graph is a coloring of its edges such that any two edges at distance at most ℓ on a boundary walk of any face receive distinct colors. One can see that this is a more restricted version of the k-facial coloring by taking the medial graph $M(G)$ of a plane graph G. However, there exist graphs that require $3\ell+1$ colors for an ℓ-facial edge-coloring. Therefore it is also conjectured that $3\ell+1$ colors suffice for an ℓ-facial edge-coloring of any plane graph. While the cases with $\ell = 1$ and $\ell = 2$ are already established, the general case is still wide open. In our talk, we give a short history of the topic and present a proof of the case $\ell = 3$.

This is a joint work with Borut Lužar.