Fast winning strategies for Staller in the Maker-Breaker domination game

Pakanun Dokyeesun *

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Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

Abstract

The Maker-Breaker domination game is played on a graph G by two players, called Dominator and Staller, who alternately choose a vertex that has not been played so far. Dominator wins the game if his moves form a dominating set. Staller wins if she plays all vertices from a closed neighborhood of a vertex $v \in V(G)$. Dominator's fast winning strategies were studied earlier. In this work, we concentrate on the cases when Staller has a winning strategy in the game. We introduce the invariant $\gamma'_{\mathrm{SMB}}(G)$ (resp., $\gamma_{\mathrm{SMB}}(G)$) which is the smallest integer k such that, under any strategy of Dominator, Staller can win the game by playing at most k vertices, if Staller (resp., Dominator) plays first on the graph G.

We prove some basic properties of $\gamma_{\mathrm{SMB}}(G)$ and $\gamma'_{\mathrm{SMB}}(G)$ and study the parameters' changes under some elementary operators as taking the disjoint union of graphs or deleting a cut vertex. We show that the inequality $\delta(G) + 1 \leq \gamma'_{\mathrm{SMB}}(G) \leq \gamma_{\mathrm{SMB}}(G)$ always holds and that for every three integers r, s, t with $2 \leq r \leq s \leq t$, there exists a graph G such that $\delta(G) + 1 = r$, $\gamma'_{\mathrm{SMB}}(G) = s$, and $\gamma_{\mathrm{SMB}}(G) = t$. We prove exact formulas for $\gamma'_{\mathrm{SMB}}(P_n)$ where P_n is a path of odd order and for $\gamma'_{\mathrm{SMB}}(T(n,k))$ where $T(n,k)$ is the tadpole graph obtained by an even cycle C_n and an odd path P_k by adding one edge.

*Email: pakanun.dokyeesun@student.fmf.uni-lj.si