Everything you always wanted to know about Long $\mathbb{C}^n$

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A complex manifold X of dimension n is said to be a *Long* $\mathbb{C}^n$ if it is the union of an increasing sequence of domains $X_1 \subseteq X_2 \subseteq X_3 \subseteq \cdots \subseteq \bigcup_{j=1}^{\infty} X_j = X$ such that each X_j is biholomorphic to the complex Euclidean space $\mathbb{C}^n$. It is immediate that any Long $\mathbb{C}$ is biholomorphic to $\mathbb{C}$. However, for $n > 1$, this class of complex manifolds is still very mysterious. In this talk I will explain what do we know about these manifolds and what are some of the main open questions. For example we know there exists a Long $\mathbb{C}^n$ which is not a Stein manifold and therefore it is not biholomorphic to $\mathbb{C}^n$. On the other hand we do not know if complex Euclidean spaces are the only examples of Long $\mathbb{C}^n$ which are Stein.

References

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