The (l, r)-Lah numbers

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After binomial coefficients, the *Stirling numbers of the first and second kind* are probably the best known triangular arrays of natural numbers, arising in many combinatorial, algebraic, and even analytic contexts. Recognized at least since the 18th century, they were joined and nicely complemented by the *Lah numbers* in the 1950s. In the 1980s, a third parameter was added to the definition of the Stirling numbers, resulting in the *r-Stirling numbers of the first and second kind*. In the last decade, the *r-Lah numbers* were defined and their properties explored. In 2021, a fourth parameter was added to the definition of the r-Stirling numbers, yielding the *(l, r)-Stirling numbers of the first and second kind*.

The problem arises: is there a generalization of the Lah numbers, analogous to the (l, r)-Stirling numbers of both kinds? Due to similar propeties of the Stirling and Lah numbers, such a generalization exists.

In this seminar we give the definition of the (l, r)-Lah numbers (denoted by $\left\lfloor {n \atop k} \right\rfloor_r^{(l)}$) and prove the recurrence relation that they satisfy. We also express them explicitly as a multiple sum. We study the difference-differential equations satisfied by their column and row generating functions and finally prove some special cases.