The distance function on Coxeter like graphs

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Abstract

Let $S_n(\mathbb{F}_2)$ be the set of all $n \times n$ symmetric matrices with coefficients from the binary field $\mathbb{F}_2 = \{0, 1\}$, and let $SGL_n(\mathbb{F}_2)$ be the subset of all invertible matrices. Let $\hat{\Gamma}_n$ be the graph with the vertex set $S_n(\mathbb{F}_2)$, where two matrices $A, B \in S_n(\mathbb{F}_2)$ form an edge if and only if $\operatorname{rank}(A - B) = 1$. Let Γ_n be the subgraph in $\hat{\Gamma}_n$, which is induced by the set $SGL_n(\mathbb{F}_2)$. In particular, Γ_3 is well known-Coxeter graph.
The distance function on $\hat{\Gamma}_n$ is given by

$$d_{\hat{\Gamma}_n}(A, B) = \begin{cases} \operatorname{rank}(A - B), & \text{if } A - B \text{ is nonalternate or zero,} \\ \operatorname{rank}(A - B) + 1, & \text{if } A - B \text{ is alternate and nonzero.} \end{cases}$$

Even the Coxeter graph shows that the distance in Γ_n must be different. The primary goal is to characterize the distance function on this graph, from which consequently, we can determine the diameter of the graph Γ_n. Since the vertices of Γ_n are invertible symmetric matrices, which are not closed under addition, describing these results is more difficult compared to the graph $\hat{\Gamma}_n$. As Γ_n is an induced subgraph in $\hat{\Gamma}_n$, we have $d_{\Gamma_n} \geq d_{\hat{\Gamma}_n}$.

Keywords: Coxeter graph,symmetric matrices, invertible symmetric matrices, rank, distance in graphs.