

# Alpe-Adria Seminar

8th meeting  
Graz

October 19, 2024

## Schedule

10:00–10:25 Amr Ali Al-Maktry: *Polynomial functions on a class of finite non-commutative rings*

10:30–10:55 Andoni Zozaya: *Verbal problems in profinite groups*

11:00–11:25 Filip Najman: *Modular curves*

*Coffee break*

12:00–12:25 Magdaléna Tinková: *Artin twin primes.*

12:30–12:55 Roman Drnovšek: *Positive Commutators on Banach lattices*

13:00–13:25 Kristina Ana Škreb: *Dimensionless  $L^p$  estimates for the Riesz vector*

14:00– Lunch at Dionysos (<http://www.restaurant-dionysos.at>)

## Location

*All talks will be in HS A in Kopernikusgasse 24 (first floor). The precise address is:*

Kopernikusgasse 24  
Graz University of Technology (TU Graz)  
8020 Graz, Austria

*The address of the restaurant is:*

Restaurant Dionysos  
Färbergasse 6  
8010 Graz, Austria

# Abstracts

## Polynomial functions on a class of finite non-commutative rings

Amr Ali Al-Maktry (TU Graz)

Let  $R$  be a finite non-commutative ring with  $1 \neq 0$ . By a polynomial function on  $R$ , we mean a function  $F: R \rightarrow R$  induced by a polynomial  $f = \sum_{i=0}^n a_i x^i \in R[x]$  via right substitution of the variable  $x$ , i.e.  $F(a) = f(a) = \sum_{i=0}^n a_i a^i$  for every  $a \in R$ . In this paper, we study the polynomial functions of the free  $R$ -algebra with a central basis  $\{1, \beta_1, \dots, \beta_k\}$  ( $k \geq 1$ ) such that  $\beta_i \beta_j = 0$  for every  $1 \leq i, j \leq k$ ,  $R[\beta_1, \dots, \beta_k]$ . Our investigation revolves around assigning a polynomial  $\lambda_f(y, z)$  over  $R$  in non-commutative variables  $y$  and  $z$  to each polynomial  $f$  in  $R[x]$ ; and describing the polynomial functions on  $R[\beta_1, \dots, \beta_k]$  through the polynomial functions induced on  $R$  by polynomials in  $R[x]$  and by their assigned polynomials in the non-commutative variables  $y$  and  $z$ . By extending results from the commutative case to the non-commutative scenario, we demonstrate that several properties and theorems in the commutative case can be generalized to the non-commutative setting with appropriate adjustments.

## Positive Commutators on Banach lattices

Roman Drnovšek (University of Ljubljana)

We will present several results on positive commutators of positive operators on Banach lattices. We will start with the following finite-dimensional theorem. Let  $A$  and  $B$  be non-negative matrices such that the commutator  $C = AB - BA$  is non-negative as well. Then, up to similarity with a permutation matrix,  $C$  is a strictly upper triangular matrix, and so it is nilpotent.

## Modular curves

Filip Najman (University of Zagreb)

Modular curves are moduli spaces of elliptic curves with prescribed images of their Galois representations. They are a key tool in studying torsion groups, isogenies and, more generally, Galois representations of elliptic curves. In recent years

great progress, in many directions, has been made in our understanding of points on modular curves of low degree. In this talk I will describe some of these recent results, their consequences, as well as open problems in the field.

### **Dimensionless $L^p$ estimates for the Riesz vector**

**Kristina Ana Škreb (University of Zagreb)**

We present a fundamentally new proof of the dimensionless  $L^p$  boundedness of the Bakry Riesz vector on manifolds with bounded geometry. The key ingredients of the proof are sparse domination and probabilistic representation of the Riesz vector. This type of proof has the significant advantage that it allows for a much stronger conclusion, giving us a new range of weighted estimates. The talk is based on joint work with K. Domelevo and S. Petermichl.

### **Artin twin primes**

**Magdaléna Tinková (TU Graz / Czech Technical University)**

In this talk, we will join two famous conjectures for primes, namely Artin's conjecture on primitive roots and the Hardy–Littlewood two-tuple conjecture. In particular, we will study the existence and the asymptotic behavior of the number of prime pairs  $p$  and  $p + d$  with the same prescribed primitive root. This is joint work with Ezra Waxman and Mikuláš Zindulka..

### **Verbal problems in profinite groups**

**Andoni Zozaya (University of Ljubljana)**

Given a group  $G$  and a group word  $w$  on  $k$  variables, we can define a map  $w: G^k \rightarrow G$  through substitution. Verbal problems study the relation between the image of this map and the subgroup it generates within  $G$ . In particular, we will study verbal conciseness – whether the subgroup is finite when the image of the map is finite – and strong verbal conciseness – whether the subgroup is finite when the image is countable – in the context of profinite groups. We will introduce a new family of verbally concise groups, the so-called analytic groups; and, based on recent work with de las Heras, we will show that several classes of profinite verbally concise groups are, in fact, strongly verbally concise.