RECENT DEVELOPMENTS IN COMBINATORIAL INVERSE SEMIGROUP THEORY

IGOR DOLINKA

Abstract

In early 1930s, Wilhelm Magnus proved his famous and celebrated result that the word problem is decidable for all one-relator groups: given two words u, v over the alphabet $A \cup A^{-1}$, there is an algorithm deciding whether u and v represent the same element of the group given by a presentation of the form $\langle A \,|\, w = 1 \rangle$. This early major result of combinatorial group theory motivated the consideration of the semigroup-theoretical analogue of the Magnus problem: Is the word problem decidable for all one-relator monoids, given by presentations of the form $\langle A \,|\, u = v \rangle$ (where $u, v \in A^*$) ? Despite almost a century of great efforts, this problem still remains, in its full generality, open.

Between the 60s and the 80s of the XX century, S.I.Adian and his collaborators solved a significant chunk of special cases of the one-relator monoid problem, thus reducing it to two special cases with include exclusively right cancellative monoids. In 2001, Margolis and Meakin published a seminal paper establishing a connection between the Adian problem and one-relator (special) *inverse* monoids. This, together with the work of their student Stephen, gave a vital impetus to the development of the combinatorial inverse semigroup theory, in particular to the study of finitely presented special inverse monoids.

In my talk I am going to provide a detailed historical account of these results and connections, and, time permitting, give an overview of recent results in the area, in particular:

- the surprising 2020 result of R.D.Gray exhibiting an example of a one-relator special inverse monoid with undecidable word problem;
- the general connection between the word problem in special inverse monoids and the decidability of the so-called *prefix membership problem* in groups, as well as a series of results, published by R.D.Gray and the speaker in 2021, establishing cases in which the latter problem is decidable;
- the recent results, also with R.D.Gray, providing (among other things) a full description of the class of all monoids arising as prefix monoids of finitely presented groups;
- even more recent (still unpublished) results concerning the right units of finitely presented special inverse monoids.

Department of Mathematics and Informatics, University of Novi Sad, Trg Dositeja Obradovića 4, 21101 Novi Sad, Serbia

Email address: `dockie@dmi.uns.ac.rs`