Alpe-Adria Seminar

(9th meeting)

May 24, 2025

Department of Mathematics
Faculty of Mathematics and Physics
University of Ljubljana
Jadranska ulica 21
10000 Ljubljana, Slovenia

Schedule

All talks will be in Lecture Room P.02, ground floor

10:00–10:25 Polona Oblak: *Exploring the extremal eigenvalue multiplicities of matrices of a given pattern*

10:30–10:55 Aleksandar Bulj: *Powers of unimodular homogeneous multipliers*

11:00–11:25 Balint Rago: *On algebraic properties of power monoids*

Coffee break

12:00–12:25 Petr Siegl: *Local form subordination without a power decay and the Riesz property of spectral projections*

12:30–12:55 Roman Bessonov: *In search of a non-factorable operator*

13:00–13:25 Nina Kamčev: *Counting regular hypergraphs*

14:00– Lunch at the restaurant Pod Vrbo, Ziherlova ulica 36, 1000 Ljubljana

Abstracts

In search of a non-factorable operator

Roman Bessonov

University of Ljubljana

A family of subspaces in a separable Hilbert space H is called chain if it is totally ordered by inclusion. Operators that leave invariant subspaces from a given chain L are called upper-triangular with respect to L. A linear positive operator T on H is said to admit a triangular factorization with respect to L if T can be represented in the form $T = T_1^* T_2$, where T_1, T_2 are upper-triangular with respect to L.

The famous result by D.Larson (1985) says that every positive bounded invertible operator on H admits a triangular factorization with respect to a chain L if and only if L is countable.

Larson's theorem implies, in particular, that there exist a non-factorable positive bounded operator with respect to the chain of subspaces $L^2[0, r]$, $r > 0$, of the space $L^2[0, +\infty)$. In 1994, L. Sakhnovich posed the problem of constructing an explicit example of such an operator. In particular, he asked if such an operator can be found among classical Wiener-Hopf convolution operators. We prove that this is not the case: every positive bounded invertible Wiener-Hopf operator admits triangular factorization with respect to the chain of subspaces $L^2[0, r]$, $r > 0$. The result complements the classical Wiener-Hopf method that implies existence of a triangular factorization with respect to the chain of subspaces $L^2[r, \infty]$, $r > 0$.

Powers of unimodular homogeneous multipliers

Aleksandar Bulj

University of Zagreb

We study asymptotically sharp estimates for the $L^p \to L^p$ norms of multipliers associated with unimodular homogeneous symbols of degree 0, i.e. multipliers associated with symbols $\xi \mapsto \exp(i\lambda\Phi(\xi/|\xi|))$, where λ is a real number and $\Phi \in C^\infty(S^{n-1})$. We show that the powers of a generic multiplier in that class exhibit asymptotically maximal order of growth. As a consequence, we disprove Maz'ya's conjecture regarding the asymptotically sharp

estimates of such multipliers in all dimensions and solve the problem posed by Dragicevic, Petermichl, and Volberg concerning the sharp lower estimate of a certain multiplier falling within the mentioned class.

Counting regular hypergraphs

Nina Kamčev

University of Zagreb

A d-regular k-uniform hypergraph is a pair (V, E), where E is a family of k-element subsets of V such that each vertex $v \in V$ is contained in exactly d elements of E. A 2-uniform hypergraph is just a graph. How many d-regular k-uniform hypergraphs are there on n vertices? We provide an asymptotic formula covering a wide range of parameters, including regular k-uniform hypergraphs of density up to ϵk^{-1}. We will discuss the perturbation method, originally applied to the graph enumeration problem by Liebenau and Wormald.

This is joint work with Anita Liebenau and Nick Wormald.

Exploring the extremal eigenvalue multiplicities of matrices of a given pattern

Polona Oblak

University of Ljubljana

The spectral properties of matrices with prescribed patterns have received significant attention in recent research. In this talk, we explore the family $S(G)$ of real symmetric matrices whose off-diagonal zero-nonzero pattern matches the one of the adjacency matrix of a given simple graph G. Finding spectra of such matrices is well-known as the Inverse Eigenvalue Problem for a Graph.

We focus on possible multiplicities of eigenvalues that matrices in $S(G)$ can achieve. It is well known that $S(G)$ contains a matrix with all eigenvalues simple, but understanding constraints on matrices with high multiplicities of eigenvalues is challenging. We will present some recent results on the maximum achievable multiplicity of any eigenvalue and on the minimal number

of distinct eigenvalues attainable by matrices in $S(G)$.

On algebraic properties of power monoids

Balint Rago

University of Graz

Let S be a multiplicatively written semigroup. The family $\mathcal{P}(S)$ of non-empty subsets of S endowed with the binary operation of setwise multiplication

$$(X, Y) \mapsto \{xy : x \in X, y \in Y\}$$

induced by S, is called the *large power semigroup* of S. Although power semigroups were already introduced in the 1950s, the study of arithmetical and algebraic properties of these objects received a lot of attention in the last decade. In this talk, we give a survey of recent developments in this area and present some new results involving the *reduced finitary power monoid* $\mathcal{P}_{\text{fin},1}(H)$ of a monoid H, which consists of all finite subsets of H containing the identity element.

To give an example, a central question, called the *isomorphism problem*, is whether an isomorphism between $\mathcal{P}_{\text{fin},1}(H_1)$ and $\mathcal{P}_{\text{fin},1}(H_2)$ for two monoids H_1 and H_2 implies that H_1 and H_2 are isomorphic. This question was answered in the negative for arbitrary monoids, yet remained open for *cancellative* monoids. We give sufficient conditions for commutative, cancellative monoids to satisfy the property above and show that there are 'well-behaved' monoids for which the isomorphism problem has a negative answer.

Local form subordination without a power decay and the Riesz property of spectral projections

Petr Siegl

TU Graz

We revisit the local form subordination condition on the perturbation of a self-adjoint operators with compact resolvent. This condition relates the size of gaps between the unperturbed eigenvalues with the strength of

perturbation and it is used to establish the Riesz basis property of the eigensystem of the perturbed operator. Our new approach allows for a slow and non-monotone decay in the subordination condition as well as for a general behavior of unperturbed eigenvalues. The abstract results are applied in Schroedinger operators with possibly unbounded or singular complex potential perturbations.

The talk is based on joint works with B. Mityagin (OSU, USA).