JORDAN $\{g,h\}$-DERIVATIONS ON TENSOR PRODUCTS OF ALGEBRAS

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ABSTRACT. Let A be a unital algebra over a field $\mathbb{F}$ with $\mathrm{char}(\mathbb{F}) \neq 2$, and let $f,g,h : A \to A$ be linear maps. We say that f is a $\{g,h\}$-derivation if $f(xy) = g(x)y + xh(y) = h(x)y + xg(y)$ for all $x,y \in A$, and we say that f is a Jordan $\{g,h\}$-derivation if $f(x \circ y) = g(x) \circ y + x \circ h(y)$ for all $x,y \in A$ (here, $x \circ y = xy + yx$). We show that if the property that every Jordan $\{g,h\}$-derivation is a $\{g,h\}$-derivation holds in A, then so does in the algebra $A \otimes S$ for every commutative unital algebra S. We also show that every semiprime algebra A has this property. Combining these two results it follows, in particular, that the classical Jordan derivations are derivations on the tensor product between a semiprime and a commutative algebra.

1. Introduction

In the recent paper [2] it was shown that, roughly speaking, if functional identities can be controlled on an algebra A, then they can be also controlled on the tensor product $A \otimes S$ of A with an arbitrary finite dimensional unital algebra S (for some identities even the assumption on finite dimensionality can be removed). As a consequence of this general result one infers that certain Jordan and Lie maps on $A \otimes S$ are of standard forms, as long as A is a well-behaved algebra with respect to functional identities. Since S plays only a formal role in these statements, one may wonder whether some simpler but more general phenomenon is hiding behind this. If we take the statement that certain Jordan or Lie maps are always standard on A as an assumption, does it follow that these maps are are also standard on $A \otimes S$? Some restrictions must certainly be imposed here for the answer is obviously negative even when A is 1-dimensional. We will assume that S is commutative. Then the question makes sense.

The purpose of this short non-technical paper is just to touch on this question, and consider only Jordan derivations which are the easiest to study among various Jordan and Lie maps. As a matter of fact, in order to obtain a positive answer we will have to deal with certain more general maps which we call Jordan $\{g,h\}$-derivations; see Section 2. In Section 3 we show that if Jordan $\{g,h\}$-derivations are "standard" on A (i.e., they are what we call $\{g,h\}$-derivations), then the same is true on $A \otimes S$. Finally, in Section 4 we show that every semiprime algebra A has

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this property. As a corollary we then obtain that the usual Jordan derivations are necessarily derivations on the tensor product of a semiprime and a commutative algebra.

Throughout the paper we assume, without further mention, that all our algebras are unital algebras over a fixed field $\mathbb{F}$ with $\mathrm{char}(\mathbb{F}) \neq 2$. By A we always denote an algebra with center Z.

2. $\{g,h\}$-derivations and Jordan $\{g,h\}$-derivations

Let $f, g, h : A \to A$ be linear maps. We will say that f is a $\{g,h\}$-*derivation* if

$$f(xy) = g(x)y + xh(y) = h(x)y + xg(y) \quad \text{for all } x, y \in A. \tag{2.1}$$

If $f = g = h$ then f is, of course, the usual *derivation*. Actually, $\{g,h\}$-derivations are very close to derivations. Namely, taking $y = 1$ in (2.1) we obtain

$$f(x) = g(x) + xh(1) = h(x) + xg(1),$$

and taking $x = 1$ we obtain

$$f(y) = g(1)y + h(y) = h(1)y + g(y).$$

Comparing both expressions we see that $g(1)$ and $h(1)$ lie in Z. Setting $\lambda = f(1) + g(1)$ we then infer from (2.1) that $d(x) = f(x) - \lambda x$ is a derivation. Thus, every $\{g,h\}$-derivation f can be written as

$$f(x) = \lambda x + d(x) \quad \text{for all } x, y \in A, \tag{2.2}$$

where d is a derivation and λ is a central element. Conversely, if f is of the form (2.2), then f is a $\{g,h\}$-derivation with, for example, $g = f$ and $h = d$. Note that g and h are not unique; we can pick any $\mu \in Z$ and take $g(x) = f(x) + \mu x$, $h(x) = d(x) - \mu x$ (choosing $\mu = -\frac{\lambda}{2}$ we thus have $g = h$). In view of these observations, the term $\{g,h\}$-derivation may now seem a bit awkward. However, it is convenient for our purposes.

As usual, we will write $x \circ y = xy + yx$ for $x, y \in A$. We call $x \circ y$ the Jordan product of x and y. Recall that the linear space of A endowed with the Jordan product is a Jordan algebra. As above, let $f, g, h : A \to A$ be linear maps. We will say that f is a *Jordan* $\{g,h\}$-*derivation* if

$$f(x \circ y) = g(x) \circ y + x \circ h(y) \quad \text{for all } x, y \in A. \tag{2.3}$$

Since the Jordan product is commutative, the symmetric identity

$$f(x \circ y) = h(x) \circ y + x \circ g(y)$$

follows from (2.3). In the special case where $f = g = h$, f is called a *Jordan derivation*. It is a classical question in which algebras (and rings) A a Jordan derivation is necessarily a derivation. In 1957, Herstein [6] showed that this is true if A is prime, and in 1975 Cusack [5] generalized this to the semiprime case. The question is still an active area of research; see, for example, the recent papers [1, 7] and references therein. We will consider a more general question whether a Jordan $\{g,h\}$-derivation is a $\{g,h\}$-derivation (note that the converse is trivially true). As will be evident from the arguments in the next section, this question naturally appears in the context of tensor products of algebras, even if one only wishes to study the ordinary Jordan derivations. Let us point out that our question is truly more general. That is, if every Jordan $\{g,h\}$-derivation of A is a $\{g,h\}$-derivation (for any pair of linear maps g and h), then every Jordan derivation of A is, of course, a derivation, while the converse is not true, as the next example shows.

Example 2.1. Suppose A contains a noncentral element a such that

$$[a,[x,y]] = 0 \quad \text{for all } x,y \in A. \tag{2.4}$$

Writing (2.4) as

$$(a \circ x) \circ y - (a \circ y) \circ x = 0$$

we see that 0 is a Jordan $\{g,-g\}$-derivation where $g(x) = a \circ x$. However, 0 is not a $\{g,-g\}$-derivation. If it was, we would have $0 = g(x)1 - xg(1) = [a,x]$ for every $x \in A$, contradicting the assumption that $a \notin Z$.

There exist algebras A containing noncentral elements a satisfying (2.4) in which not every Jordan derivation is a derivation [3, Example 4.2]. However, there also exist such in which all Jordan derivations are derivations. For instance, take the algebra A of all 3×3 upper triangular matrices whose diagonal entries are equal to each other. Note that A is noncommutative, yet every commutator $[x,y]$ with $x,y \in A$ lies in Z. The condition from the preceding paragraph is thus fulfilled, so not every Jordan $\{g,h\}$-derivation of A is a $\{g,h\}$-derivation. However, every Jordan derivation of A is a derivation. One can check this by a direct computation, that is, by considering the action of a Jordan derivation on $e_{12}\circ e_{12} = 0$, $e_{23}\circ e_{23} = 0$, and $e_{12}\circ e_{23} = e_{13}$ (here, e_{ij} are matrix units). We leave details to the reader.

Let us add a similar, but in some sense more convincing example of a Jordan $\{g,h\}$-derivation which is not even a $\{g',h'\}$-derivation for any pair of linear maps g' and h'.

Example 2.2. Assume now that A contains a noncentral element a satisfying $[a,x] \in Z$ for every $x \in A$ (using the Jacobi identity we see that this assumption implies (2.4)). Then $f(x) = a\circ x$ is a Jordan $\{f,0\}$-derivation, but is not a $\{g',h'\}$-derivation for any linear maps g' and h'. If it was, f would be of the form (2.2) for some derivation d and $\lambda \in Z$, yielding a contradiction $2a = f(1) = \lambda$ (namely, $d(1) = 0$ for $d(1) = d(1^2) = d(1)1 + 1d(1) = 2d(1)$).

Since there are many generalizations of the concept of a derivation, let us conclude this section with a few words of clarification. One should not confuse the notion of a $\{g,h\}$-derivation with that of a (g,h)-derivation where g and h are automorphisms of A. On the other hand, $\{g,h\}$-derivations are special examples of what is known in the literature as generalized derivations. The notion of a Jordan $\{g,h\}$-derivation, however, is a generalization of that of a Jordan generalized derivation from [8].

3. Jordan $\{g,h\}$-derivations on tensor products

We will now show that the general question posed in Section 1 has an affirmative answer for Jordan $\{g,h\}$-derivations.

Theorem 3.1. *If an algebra A has the property that every Jordan $\{g,h\}$-derivation of A is a $\{g,h\}$-derivation, then the algebra $A\otimes S$, where S is an arbitrary commutative algebra, has the same property.*

Proof. Take a basis $\{b_t\,|\,t\in T\}$ of S. Let f be a Jordan $\{g,h\}$-derivation of $A\otimes S$. We can write

$$f(u)=\sum_{t\in T}f_t(u)\otimes b_t,\quad g(u)=\sum_{t\in T}g_t(u)\otimes b_t,\quad h(u)=\sum_{t\in T}h_t(u)\otimes b_t$$

where for each $u\in A\otimes S$ we have $f_t(u)=g_t(u)=h_t(u)=0$ for all but finitely many $t\in T$. Take $x,y\in A$ and $r,s\in S$. By assumption,

$$f\big((x\circ y)\otimes rs\big)=f\big((x\otimes r)\circ(y\otimes s)\big)=g(x\otimes r)\circ(y\otimes s)+(x\otimes r)\circ h(y\otimes s),$$

that is,

$$\sum_{t\in T}f_t\big((x\circ y)\otimes rs\big)\otimes b_t=\sum_{w\in T}\big(g_w(x\otimes r)\circ y\big)\otimes b_w s+\sum_{w\in T}\big(x\circ h_w(y\otimes s)\big)\otimes b_w r.$$

Fix r and s, and take $\alpha_{tw},\beta_{tw}\in\mathbb{F}$ such that

$$b_w r=\sum_{t\in T}\alpha_{tw}b_t,\quad b_w s=\sum_{t\in T}\beta_{tw}b_t.$$

The right-hand side of the last identity can then be written as

$$\sum_{t\in T}\left(\Big(\sum_{w\in T}\beta_{tw}g_w(x\otimes r)\Big)\circ y\right)\otimes b_t+\sum_{t\in T}\left(x\circ\Big(\sum_{w\in T}\alpha_{tw}h_w(y\otimes s)\Big)\right)\otimes b_t,$$

which yields

$$f_t\big((x\circ y)\otimes rs\big)=\Big(\sum_{w\in T}\beta_{tw}g_w(x\otimes r)\Big)\circ y+x\circ\Big(\sum_{w\in T}\alpha_{tw}h_w(y\otimes s)\Big).$$

Fixing $t\in T$ we thus see that the map $\tilde f(x)=f_t\big(x\otimes rs\big)$ is a Jordan $\{\tilde g,\tilde h\}$-derivation of A, where

$$\tilde g(x)=\sum_{w\in T}\beta_{tw}g_w(x\otimes r),\quad \tilde h(y)=\sum_{w\in T}\alpha_{tw}h_w(y\otimes s).$$

According to our assumption $\tilde f$ is then a $\{\tilde g,\tilde h\}$-derivation, so that

$$f_t(xy\otimes rs)=\Big(\sum_{w\in T}\beta_{tw}g_w(x\otimes r)\Big)y+x\Big(\sum_{w\in T}\alpha_{tw}h_w(y\otimes s)\Big)$$

for all $x, y \in A$ and for every $t \in T$. Accordingly,

$$\begin{aligned}
&f(x \otimes r \cdot y \otimes s) = f(xy \otimes rs) = \sum_{t\in T} f_t(xy \otimes rs) \otimes b_t \\
=&\sum_{t\in T}\Big(\sum_{w\in T}\beta_{tw} g_w(x\otimes r)\Big) y \otimes b_t + \sum_{t\in T} x\Big(\sum_{w\in T}\alpha_{tw} h_w(y\otimes s)\Big)\otimes b_t \\
=&\Big(\sum_{w\in T} g_w(x\otimes r)\Big) y \otimes \Big(\sum_{t\in T}\beta_{tw} b_t\Big) + x\Big(\sum_{w\in T} h_w(y\otimes s)\Big)\otimes\Big(\sum_{t\in T}\alpha_{tw} b_t\Big) \\
=&\sum_{w\in T} g_w(x\otimes r) y \otimes b_w s + \sum_{w\in T} x h_w(y\otimes s)\otimes b_w r \\
=&g(x\otimes r)\cdot y\otimes s + x\otimes r\cdot h(y\otimes s).
\end{aligned}$$

This shows that $f(uv) = g(u)v + uh(v)$ for all $u, v \in A \otimes S$. Since f is a Jordan $\{g, h\}$-derivation, the identity $f(uv) = h(u)v + ug(v)$ clearly follows. □

4. Jordan $\{g, h\}$-derivations on semiprime algebras

In view of Example 2.1, our problem makes sense only in algebras not containing noncentral elements a satisfying (2.4). In this framework it can be presented in a simpler form.

Lemma 4.1. *Let A be an algebra in which only central elements a satisfy* (2.4)*. If every Jordan $\{g, g\}$-derivation of A is a $\{g', h'\}$-derivation (for some linear maps g' and h'), then every Jordan $\{g, h\}$-derivation of A is a $\{g, h\}$-derivation.*

Proof. Let f be a Jordan $\{g, h\}$-derivation. From $f(x \circ y) = g(x) \circ y + x \circ h(y)$ we get

$$f(x) = g(x) + x \circ a,$$

where $a = \frac{1}{2}h(1)$, and

$$f(y) = h(y) + y \circ b$$

where $b = \frac{1}{2}g(1)$. Accordingly,

$$f(x \circ y) = f(x) \circ y + x \circ f(y) - (x \circ a) \circ y - x \circ (y \circ b). \tag{4.1}$$

Since x and y occur symmetrically in $f(x \circ y) - f(x) \circ y - x \circ f(y)$, it follows that

$$(x \circ a) \circ y + x \circ (y \circ b) = (y \circ a) \circ x + y \circ (x \circ b),$$

which can be rewritten as $[a-b, [x, y]] = 0$. According to our assumption it follows that $\gamma := a - b \in Z$. Consequently, we can write (4.1) as

$$f(x \circ y) = \big(f(x) - x \circ b - \gamma x\big) \circ y + x \circ \big(f(y) - y \circ b - \gamma y\big).$$

We are now in a position to use the assumption of the lemma. Thus, there exist linear maps g', h' such that f is a $\{g', h'\}$-derivation. In particular, f is a Jordan $\{g', h'\}$-derivation; since f is also a Jordan $\{g, h\}$-derivation, it readily follows that

$$\big(g(x) - g'(x)\big) \circ y = x \circ \big(h'(y) - h(y)\big) \tag{4.2}$$

for all $x, y \in A$. Setting $y = 1$ we obtain

$$2\big(g(x) - g'(x)\big) = x \circ \big(h'(1) - h(1)\big), \tag{4.3}$$

setting $x = 1$ we obtain

$$2\big(h'(y) - h(y)\big) = \big(g(1) - g'(1)\big) \circ y, \tag{4.4}$$

and setting $x = y = 1$ we obtain

$$g(1) - g'(1) = h'(1) - h(1).$$

Denoting this element by α it follows from (4.2) that $(x \circ \alpha) \circ y = x \circ (\alpha \circ y)$. That is, $[\alpha, [x, y]] = 0$ and hence $\alpha \in Z$ by our assumption. From (4.3) it now follows that $g(x) = g'(x) + \alpha x$, and from (4.4) it follows that $h(y) = h'(y) - \alpha y$. Therefore

$$g(x)y + xh(y) = g'(x)y + xh'(y) = f(xy),$$

proving that f is a $\{g, h\}$-derivation. $\square$

Recall that an algebra A is said to be *semiprime* if it has no nonzero nilpotent ideals. Equivalently, for each $a \in A$, $aya = 0$ for every $y \in A$ implies $a = 0$. Let us first check that semiprime algebras satisfy the condition from Lemma 4.1.

Lemma 4.2. *If an element a from a semiprime algebra A satisfies* (2.4)*, then $a \in Z$.*

Proof. Since a commutes with every commutator, it commutes with $y[a, x] = [ya, x] - [y, x]a$. Consequently, $[a, y][a, x] = [a, y[a, x]] - y[a, [a, x]] = 0$ for all $x, y \in A$. Replacing y by xy and using $[a, xy] = [a, x]y + x[a, y]$ it follows that $[a, x]y[a, x] = 0$. But then $[a, x] = 0$ for A is semiprime. That is, $a \in Z$. $\square$

Let us also recall that an algebra A is said to be *prime* if the product of any of its two nonzero ideals is nonzero. Further, we say that an ideal of an algebra is prime if the corresponding factor algebra is prime. As is well-known, the intersection of all prime ideals of a semiprime algebra is $\{0\}$. This makes it possible for one to reduce some problems on semiprime algebras to prime algebras, which are much easier to deal with. One of the reasons for this is that the center of a prime algebra can be embedded into a certain field, called the *extended centroid*, which has many extremely useful properties; see, e.g., [4, Sections 7.5 and 7.6] for a survey on this subject.

Theorem 4.3. *Every Jordan $\{g, h\}$-derivation of a semiprime algebra A is a $\{g, h\}$-derivation.*

Proof. In light of Lemmas 4.1 and 4.2, it suffices to consider a Jordan $\{g, g\}$-derivation f. The goal is to show that f is the sum of a derivation and the identity map multiplied by an element from Z (see the beginning of Section 2).

Taking $y = 1$ in

$$f(x \circ y) = g(x) \circ y + x \circ g(y) \tag{4.5}$$

we obtain

$$g(x) = f(x) + x \circ \beta \tag{4.6}$$

where $\beta = -\frac{1}{2}g(1)$. Suppose, temporarily, that $\beta \in Z$. Define $d : A \to A$ by

$$d(x) = f(x) + 4\beta x,$$

and note that

$$\begin{aligned} d(x \circ y) &= f(x \circ y) + 4\beta x \circ y = g(x) \circ y + x \circ g(y) + 4\beta x \circ y \\ &= \big(d(x) - 2\beta x\big) \circ y + x \circ \big(d(y) - 2\beta y\big) + 4\beta x \circ y \\ &= d(x) \circ y + x \circ d(y). \end{aligned}$$

That is, d is a Jordan derivation, and hence a derivation by [5]. Since $f(x) = d(x) - 4\beta x$, this completes the proof. Thus, all we have to show is that β lies in the center.

Returning to (4.5) and (4.6), we have

$$f(x \circ y) = f(x) \circ y + x \circ f(y) + (x \circ \beta) \circ y + x \circ (y \circ \beta). \tag{4.7}$$

Recall the Jordan identity $(x^2 \circ y) \circ x = x^2 \circ (y \circ x)$. Using (4.7) we will now consider the action of f on each of the two sides, and thereby derive a certain identity involving β. Firstly, we have

$$\begin{aligned} f((x^2 \circ y) \circ x) =& f(x^2 \circ y) \circ x + (x^2 \circ y) \circ f(x) \\ &+ ((x^2 \circ y) \circ \beta) \circ x + (x^2 \circ y) \circ (x \circ \beta). \end{aligned}$$

Since

$$\begin{aligned} f(x^2 \circ y) =& f(x^2) \circ y + x^2 \circ f(y) + (x^2 \circ \beta) \circ y + x^2 \circ (y \circ \beta) \\ =& (f(x) \circ x) \circ y + ((x \circ \beta) \circ x) \circ y + x^2 \circ f(y) \\ &+ (x^2 \circ \beta) \circ y + x^2 \circ (y \circ \beta), \end{aligned}$$

it follows that

$$\begin{aligned} f((x^2 \circ y) \circ x) =& ((f(x) \circ x) \circ y) \circ x + (((x \circ \beta) \circ x) \circ y) \circ x + (x^2 \circ f(y)) \circ x \\ &+ ((x^2 \circ \beta) \circ y) \circ x + (x^2 \circ (y \circ \beta)) \circ x + (x^2 \circ y) \circ f(x) \\ &+ ((x^2 \circ y) \circ \beta) \circ x + (x^2 \circ y) \circ (x \circ \beta). \end{aligned}$$

Secondly, we have

$$\begin{aligned} f(x^2 \circ (y \circ x)) =& f(x^2) \circ (y \circ x) + x^2 \circ f(y \circ x) \\ &+ (x^2 \circ \beta) \circ (y \circ x) + x^2 \circ ((y \circ x) \circ \beta) \\ =& (f(x) \circ x) \circ (y \circ x) + ((x \circ \beta) \circ x) \circ (y \circ x) + x^2 \circ (f(y) \circ x) \\ &+ x^2 \circ (y \circ f(x)) + x^2 \circ ((y \circ \beta) \circ x) + x^2 \circ (y \circ (x \circ \beta)) \\ &+ (x^2 \circ \beta) \circ (y \circ x) + x^2 \circ ((y \circ x) \circ \beta). \end{aligned}$$

Equating both expressions, and then expanding and collecting the terms we obtain

$$\begin{aligned} & x^2 y[\beta, x] - xy[\beta, x^2] + y\big([\beta, x]x^2 - x[\beta, x]x + x^2[\beta, x]\big) \\ =& [\beta, x]yx^2 - [\beta, x^2]yx + \big([\beta, x]x^2 - x[\beta, x]x + x^2[\beta, x]\big)y \end{aligned} \tag{4.8}$$

for all $x, y \in A$. Our intention now is to show that $\beta \in Z$ follows merely from this identity. To this end, it suffices to consider the case where A is prime. Namely, if I is an ideal of A, then $\beta + I \in A/I$ satisfies the same identity for all $x + I, y + I \in A/I$. If the desired conclusion holds for prime algebras, then it follows that $[\beta, A] \subseteq I$

for every prime ideal of A. Since the intersection of all prime ideals is $\{0\}$, this yields $[\beta, A] = \{0\}$.

Thus, assume that A is prime. If $x \in A$ is such that $1, x, x^2$ are linearly independent over the extended centroid C of A, then it follows from (4.8) that $[\beta, x]$ lies in the linear span of $1, x, x^2$ [4, Theorem 7.43]. In particular, $[[\beta, x], x] = 0$. Assume, therefore, that $x^2 = \lambda x + \mu$ for some $\lambda, \mu \in C$. Then (4.8) reduces to $(\lambda^2 + 4\mu)[[\beta, x], y] = 0$ for all $y \in A$. Thus, we have $[[\beta, x], x] = 0$ in this case too, unless $\lambda^2 + 4\mu = 0$. If the latter holds, then $(x - \frac{1}{2}\lambda)^2 = 0$. Writing $x' = x - \frac{1}{2}\lambda$ we thus have

$$[[[\beta, x], x], x] = [[[\beta, x'], x'], x'] = \beta x'^3 - 3x'\beta x'^2 + 3x'^2\beta x' - x'^3\beta = 0$$

for $x'^2 = 0$. Thus, $[[[\beta, x], x], x] = 0$ for all $x \in A$. But this forces $[\beta, x] = 0$ for all $x \in A$ by, for example, [9, Theorem 1]. Thus, $\beta \in Z$. $\square$

Theorems 3.1 and 4.3 together show that every Jordan $\{g, h\}$-derivation of the tensor product of a semiprime and a commutative algebra is a $\{g, h\}$-derivation. As a particular case we have the following corollary.

Corollary 4.4. *Let A be a semiprime and S be a commutative algebra. Then every Jordan derivation of $A \otimes S$ is a derivation.*

Let us point out that $A \otimes S$ is not semiprime if S is not semiprime. On the other hand, even the tensor product of semiprime algebras is not always semiprime. Corollary 4.4 is therefore, to the best of our knowledge, a new result.

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