

Holomorphic families of Fatou-Bieberbach domains and applications to Oka manifolds

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Abstract We construct holomorphically varying families of Fatou-Bieberbach domains with given centres in the complement of any compact polynomially convex subset K of \mathbb{C}^n for $n > 1$. This provides a simple proof of the recent result of Y. Kusakabe to the effect that the complement $\mathbb{C}^n \setminus K$ of any polynomially convex subset K of \mathbb{C}^n is an Oka manifold. The analogous result is obtained with \mathbb{C}^n replaced by any Stein manifold with the density property.

Keywords Fatou-Bieberbach domain, polynomially convex set, Oka manifold

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1. Introduction

A *Fatou-Bieberbach domain* in \mathbb{C}^n is a proper subdomain $\Omega \subsetneq \mathbb{C}^n$ which is biholomorphic to \mathbb{C}^n . No such domains exists for $n = 1$, but they are plentiful for any $n > 1$; see the survey of this topic in [3, Chapter 4]. In particular, the basin of attraction of an attracting fixed point of a holomorphic automorphism of \mathbb{C}^n (or in fact of any complex manifold) is biholomorphic to \mathbb{C}^n , cf. [11] and [3, Theorem 4.3.2]. Furthermore, for any compact polynomially convex set $K \subset \mathbb{C}^n$ ($n > 1$) and point $p \in \mathbb{C}^n \setminus K$ there is a Fatou-Bieberbach domain $\Omega \subset \mathbb{C}^n$ such that $p \in \Omega$ and $K \cap \Omega = \emptyset$; this is a special case of [5, Proposition 9] where the same is shown with p replaced by any compact convex set.

In this note we prove the following more general result in this direction.

Theorem 1.1. *Let K be a compact polynomially convex set in \mathbb{C}^n for some $n > 1$, L be a compact polynomially convex set in \mathbb{C}^N for some $N \in \mathbb{N}$, and $f : U \rightarrow \mathbb{C}^n$ be a holomorphic map on an open neighbourhood $U \subset \mathbb{C}^N$ of L such that $f(z) \in \mathbb{C}^n \setminus K$ for all $z \in L$. Then there are an open neighbourhood $V \subset U$ of L and a holomorphic map $F : V \times \mathbb{C}^n \rightarrow \mathbb{C}^n$ such that for every $z \in V$ we have that $F(z, 0) = f(z)$ and the map $F(z, \cdot) : \mathbb{C}^n \rightarrow \mathbb{C}^n \setminus K$ is injective. Hence, $\Omega_z := \{F(z, \zeta) : \zeta \in \mathbb{C}^n\}$ is a Fatou-Bieberbach domain in $\mathbb{C}^n \setminus K$ for each $z \in V$.*

A proof of this result based solely on Andersén-Lempert theory is given in Section 2; it also applies if \mathbb{C}^N is replaced by an arbitrary Stein manifold, and also to variable fibres $K_z \subset \mathbb{C}^n$, $z \in L$, with polynomially convex graph (see Remark 2.2). For a convex parameter space $L \subset \mathbb{C}^N$ we prove the analogous result with \mathbb{C}^n replaced by an arbitrary Stein manifold having the density property; see Theorem 3.1.

These two theorems immediately imply the following recent and very interesting result of Yuta Kusakabe.

Theorem 1.2. (Kusakabe, [9, Theorem 1.2 and Corollary 1.3].) *For any compact holomorphically convex subset K in a Stein manifold Y with the density property the complement $Y \setminus K$ is an Oka manifold. In particular, the complement $\mathbb{C}^n \setminus K$ of any compact polynomially convex set K in \mathbb{C}^n for $n > 1$ is an Oka manifold.*

This is the first result in the literature which gives a large class of Oka domains in \mathbb{C}^n for any $n > 1$, and it provides an affirmative answer to a long-standing problem. As noted in [9, Corollary 1.4], it follows from Theorem 1.2 and [4, Theorem 1.1] that for any compact polynomially convex set K in \mathbb{C}^n ($n > 1$), the complement $\mathbb{C}^n \setminus K$ (like any n -dimensional Oka manifold) is the image of a strongly dominating holomorphic map $\mathbb{C}^n \rightarrow \mathbb{C}^n \setminus K$.

Recall that a complex manifold Y is said to be an *Oka manifold* if every holomorphic map from a neighbourhood of a compact (geometrically) convex set L in a Euclidean space \mathbb{C}^N into Y is a uniform limit on L of entire maps $\mathbb{C}^N \rightarrow Y$ (see [3, Definition 5.4.1]; this is also called the *convex approximation property* and denoted CAP). By [3, Theorem 5.4.4], holomorphic maps $S \rightarrow Y$ from any reduced Stein space S to an Oka manifold Y satisfy all natural Oka-type properties. In his recent paper [8], Kusakabe showed that a complex manifold Y is Oka if (and only if) it satisfies the following condition:

(*) For any compact convex set $L \subset \mathbb{C}^N$, open set $U \subset \mathbb{C}^N$ containing L , and holomorphic map $f : U \rightarrow Y$ there are an open set V with $L \subset V \subset U$ and a holomorphic map $F : V \times \mathbb{C}^N \rightarrow Y$ with $F(\cdot, 0) = f|_V$ such that

$$\frac{\partial}{\partial \zeta} \Big|_{\zeta=0} F(z, \zeta) : \mathbb{C}^N \rightarrow T_{f(z)}Y \text{ is surjective for every } z \in V.$$

A map F with these properties is called a *dominating holomorphic spray over f* . This is a restricted version of condition Ell_1 introduced by Gromov [6, p. 72] (see also [7]). In [8], Kusakabe used the technique of gluing sprays from [3, Sect. 5.9] to show that this condition implies CAP, so Y is an Oka manifold. Conversely, it has been known before that every Oka manifold satisfies condition Ell_1 (see [3, Corollary 8.8.7]).

Theorem 1.1 provides a very special dominating spray with values in $\mathbb{C}^n \setminus K$ over any given holomorphic map $f : L \rightarrow \mathbb{C}^n \setminus K$, thereby proving Theorem 1.2 in the case $Y = \mathbb{C}^n$. In exactly the same way, Theorem 3.1 implies the general case of Theorem 1.2.

Kusakabe also proved in [9, Theorem 4.2] that certain closed noncompact sets in Stein manifolds Y with the density property have Oka complements. He constructed a holomorphically varying family $f(z) \in \Omega_z \subset Y \setminus K$ ($z \in L$) of nonautonomous basins with uniform bounds (i.e., basins of random sequences of automorphisms of Y which are uniformly attracting at $f(z) \in Y \setminus K$); these are elliptic manifolds as shown by Fornæss and Wold [2], hence Oka. When $Y = \mathbb{C}^n$, the domains Ω_z can be chosen Fatou-Bieberbach domains by using Theorem 1.1 with variable fibres (cf. Remark 2.2). Kusakabe's proof of [9, Theorem 4.2] can also be modified so as to provide a family of Fatou-Bieberbach domains in the general situation under consideration.

2. Proof of Theorem 1.1

We shall use some standard facts concerning polynomially convex sets; we refer the reader to the monograph by E. L. Stout [12]. Firstly, if $K_1, K_2 \subset \mathbb{C}^n$ is a pair of disjoint compact sets such that $K_1 \cup K_2$ is polynomially convex, then for any polynomially convex set $K'_1 \subset K_1$ the union $K'_1 \cup K_2$ is also polynomially convex. Secondly, every compact polynomially convex set $K \subset \mathbb{C}^n$ is the zero set of a nonnegative plurisubharmonic exhaustion function $\rho : \mathbb{C}^n \rightarrow [0, +\infty)$ which is strongly plurisubharmonic on $\mathbb{C}^n \setminus K$. Choosing a sequence $c_1 > c_2 > \dots > 0$ with $\lim_{i \rightarrow \infty} c_i = 0$ and setting $K_i = \{\rho \leq c_i\} \supset K$ yields a decreasing sequence of compact polynomially convex sets with K_{i+1} contained in the interior of K_i for every $i \in \mathbb{N}$.

Let f , K and L be as in the theorem. We replace L by a slightly bigger polynomially convex set (still denoted L) contained in U and such that $f(z) \in \mathbb{C}^n \setminus K$ for all $z \in L$. Choose a sequence $K_i \supset K$ as above, with K_1 chosen close enough to K such that $f(z) \in \mathbb{C}^n \setminus K_1$ for every $z \in L$. The compact set $L \times K_i \subset \mathbb{C}^{N+n}$ is polynomially convex for every $i \in \mathbb{N}$. Applying the change of coordinates $\psi(z, \zeta) = (z, \zeta - f(z))$ replaces f by the zero function, and for every $i \in \mathbb{N}$ the set

$$S_i = \psi(L \times K_i) \subset L \times \mathbb{C}^n \subset \mathbb{C}^{N+n}$$

is polynomially convex and does not intersect $\mathbb{C}^N \times \{0\}^n$. Hence, $(L \times \{0\}^n) \cup S_1$ is polynomially convex. Therefore, there is a small closed ball $B \subset \mathbb{C}^n$ centred at $0 \in \mathbb{C}^n$ such that $(L \times B) \cap S_1 = \emptyset$ and $(L \times B) \cup S_1$ is polynomially convex. Since $S_i \subset S_1$ is polynomially convex, it follows that $(L \times B) \cup S_i$ is polynomially convex for each $i \in \mathbb{N}$.

The following lemma will be used in the inductive construction.

Lemma 2.1. *(Assumptions as above.) Let $B' \subset \mathbb{C}^n$ be a closed ball centred at the origin with $B \subset B'$. Then, there are an open neighbourhood $U' \subset U$ of L and a biholomorphic map $\Phi : U' \times \mathbb{C}^n \rightarrow U' \times \mathbb{C}^n$ of the form $\Phi(z, \zeta) = (z, \phi(z, \zeta))$ such that*

- (a) Φ approximates the identity map as closely as desired on $L \times B$ and $\Phi(z, 0) = (z, 0)$ for all $z \in U'$,
- (b) $\Phi(S_1) \cap (L \times B') = \emptyset$, and
- (c) the set $\Phi(S_2) \cup (L \times B')$ is polynomially convex.

Proof. Choose $r > 1$ such that $rB = B'$. Letting $\theta_r(z, \zeta) = (z, r\zeta)$ for $z \in \mathbb{C}^N$ and $\zeta \in \mathbb{C}^n$, we have that $(L \times B') \cap \theta_r(S_1) = \emptyset$ and

$$(2.1) \quad (L \times B') \cup \theta_r(S_1) = \theta_r((L \times B) \cup S_1) \text{ is polynomially convex.}$$

Consider the isotopy of biholomorphic maps ϕ_t on a neighbourhood of $(L \times B) \cup S_1$ in \mathbb{C}^{N+n} for $t \in [1, r]$ which equals the identity map on a neighbourhood of $L \times B$ and equals θ_t on a neighbourhood of S_1 . Note that $\theta_t(S_1)$ is disjoint from $L \times B$ and the union $(L \times B) \cup \theta_t(S_1)$ is polynomially convex for all $t \in [1, r]$ (since it is contained in $\theta_t((L \times B) \cup S_1) = (L \times tB) \cup \theta_t(S_1)$ which is polynomially convex). Hence, by the parametric Andersén-Lempert theorem (see [10] and [3, Theorem 4.12.3]) there is a holomorphic automorphism of \mathbb{C}^{N+n} of the form $\Phi(z, \zeta) = (z, \phi(z, \zeta))$ which approximates the identity map on $L \times B$ and it approximates θ_r on S_1 . Hence, conditions (a) and (b) in the lemma hold. Assuming that the approximations are close enough, we have $\Phi(S_2) \subset \theta_r(S_1)$. Note that $\Phi(S_2)$ is polynomially convex. In view of (2.1) it follows that $\Phi(S_2) \cup (L \times B')$ is polynomially convex as well which gives condition (c). \square

Proof of Theorem 1.1. We apply the push-out method described in [3, Section 4]. Using Lemma 2.1 we inductively construct a decreasing sequence of open neighborhoods U_k of L and holomorphic automorphisms $\Phi_k(z, \zeta) = (z, \phi_k(z, \zeta))$ of $U_k \times \mathbb{C}^n$ such that, setting

$$\Phi^k = \Phi_k \circ \Phi_{k-1} \circ \cdots \circ \Phi_1 : U_k \times \mathbb{C}^n \rightarrow U_k \times \mathbb{C}^n,$$

the following conditions hold for every $k \in \mathbb{N}$.

- (i) Φ_k approximates the identity map as closely as desired on $L \times kB$ and $\Phi_k(z, 0) = (z, 0)$ for all $z \in U_k$.
- (ii) $\Phi^k(S_k) \cap (L \times (k+1)B) = \emptyset$.
- (iii) The set $\Phi^k(S_{k+1}) \cup (L \times (k+1)B)$ is polynomially convex.

Indeed, the lemma furnishes the first map Φ_1 with $B' = 2B$ and the sets $S_2 \subset S_1$; every subsequent step is of the same form by just increasing the indices. Assuming that the approximations are close enough, [3, Proposition 4.4.1 and Corollary 4.4.2] show that the limit $\Phi = \lim_{k \rightarrow \infty} \Phi^k$ exists uniformly on compacts on the domain

$$\Omega = \{(z, \zeta) \in L \times \mathbb{C}^n : \Phi^k(z, \zeta) \text{ is a bounded sequence}\} = \bigcup_{k=1}^{\infty} (\Phi^k)^{-1}(L \times kB),$$

and for every $z \in L$, $\Phi(z, \cdot)$ maps the fibre $\Omega_z = \{\zeta \in \mathbb{C}^n : (z, \zeta) \in \Omega\}$ biholomorphically onto \mathbb{C}^n . By condition (ii) the set $S = \psi(L \times K)$ does not intersect Ω (it has been pushed to infinity by the sequence Φ^k). Hence, the inverse map $\Phi^{-1}(z, \zeta) = (z, \varphi(z, \zeta))$ provides a holomorphic family of Fatou-Bieberbach maps $\varphi(z, \cdot) : \mathbb{C}^n \rightarrow \mathbb{C}^n$ ($z \in L$) such that $\varphi(z, 0) = 0$ and its image does not intersect the set $K - f(z)$. The function $F(z, \zeta) = \varphi(z, \zeta) + f(z)$ for $z \in L$ and $\zeta \in \mathbb{C}^n$ satisfies the conclusion of the theorem. \square

Remark 2.2. The above proof also applies in the case when the product $L \times K$ is replaced by a compact polynomially convex set $\tilde{K} \subset \mathbb{C}^{N+n}$ projecting onto L whose fibres K_z ($z \in L$) depend on z . The conclusion remains the same, that is, given a holomorphic map $f : L \rightarrow \mathbb{C}^n$ with $f(z) \in \mathbb{C}^n \setminus K_z$ for all $z \in L$, there is a holomorphically variable family of Fatou-Bieberbach domains $f(z) \in \Omega_z \subset \mathbb{C}^n \setminus K_z$ for all $z \in L$.

3. Fatou-Bieberbach domains in Stein manifolds with the density property

In this section we give a version of Theorem 1.1 with \mathbb{C}^n replaced by an arbitrary Stein manifold with the density property. (See Varolin [14] or [3, Definition 4.10.1] for this notion.) Every such manifold has dimension > 1 . The following result is similar to Theorem 1.1, but we impose the extra condition that the set L is geometrically convex.

Theorem 3.1. *Let X be a Stein manifold with the density property, K be a compact holomorphically convex set in X , L be a compact convex set in \mathbb{C}^N for some $N \in \mathbb{N}$, and $f : U \rightarrow X$ be a holomorphic map on an open neighbourhood $U \subset \mathbb{C}^N$ of L such that $f(z) \in X \setminus K$ for all $z \in L$. Then there are a neighbourhood $V \subset U$ of L and a holomorphic map $F : V \times \mathbb{C}^n \rightarrow X$ with $n = \dim X$ such that for every $z \in V$ we have that $F(z, 0) = f(z)$ and the map $F(z, \cdot) : \mathbb{C}^n \rightarrow X \setminus K$ is injective.*

Hence, $\Omega_z := \{F(z, \zeta) : \zeta \in \mathbb{C}^n\} \subset X \setminus K$ is a Fatou-Bieberbach domain of the first kind (i.e., biholomorphic to \mathbb{C}^n) for each $z \in V$.

The proof of Theorem 3.1 depends on the following interpolation result for graphs. We denote by dist_X a distance function on X compatible with the manifold topology.

Lemma 3.2. *Let X, K, L, U and f be as above, and let $z_0 \in L$ be arbitrary. Then for any $\epsilon > 0$ there exist a neighbourhood $V \subset U$ of L and a fibred holomorphic automorphism $\phi(z, x) = (z, \varphi(z, x))$ of $V \times X$ such that $\phi(z, f(z)) = (z, f(z_0))$ for all $z \in V$, and $\text{dist}_X(\varphi(z, x), x) < \epsilon$ for all $z \in L$ and $x \in K$.*

Proof. We may assume that $z_0 = 0 \in \mathbb{C}^N$. Let V_1, \dots, V_m be complete holomorphic vector fields on X such that $V_1(x), \dots, V_m(x)$ span the tangent space $T_x X$ for all $x \in X$ (such exist by [3, Proposition 5.6.23] since X is Stein and has the density property). Let $\psi_{1,s}, \dots, \psi_{m,s}$ denote their respective flows, $s \in \mathbb{C}$. Consider the map $\Psi : \mathbb{C}^m \times X \rightarrow X$ defined for $s = (s_1, \dots, s_m) \in \mathbb{C}^m$ and $x \in X$ by

$$\Psi(s_1, \dots, s_m, x) = \psi_{m,s_m} \circ \dots \circ \psi_{1,s_1}(x).$$

Note that $\Psi_s := \Psi(s, \cdot) \in \text{Aut}(X)$ for every $s \in \mathbb{C}^m$. Then the partial differential $\partial_s|_{s=0}\Psi(s, f(0))$ has maximal rank $n = \dim X$, so there exists an n -dimensional linear subspace $\Lambda \subset \mathbb{C}^m$ on which this differential has rank n . We may assume that $\Lambda = \mathbb{C}^n \times \{0\}^{m-n}$. Write $s = (s', s'')$ with $s' \in \mathbb{C}^n$ and $s'' \in \mathbb{C}^{m-n}$. It follows that there exists $\delta > 0$ such that the map $s' \mapsto \tilde{\Psi}_{s'} := \Psi((s', 0''), f(0)) \in X$ is an embedding of the open δ -ball $B_\delta \subset \mathbb{C}^n$ centred at $0 \in \mathbb{C}^n$ onto an open neighbourhood of $f(0) \in X$.

We replace L by a slightly larger convex set $L' \subset U$ with $L \subset (L')^\circ$ without changing the notation. We also choose a compact holomorphically convex set $K' \subset X$ containing K in its interior and such that $f(z) \in X \setminus K'$ for all $z \in L$. Set $f_t(z) = f(t \cdot z)$ for $z \in L$ and $t \in [0, 1]$. Consider the isotopy $\phi_t(z, x)$ defined to be the identity near $L \times K'$ and $\phi_t(z, f(z)) = (z, f_{1-t}(z))$, $0 \leq t \leq 1$ on the graph $Z := \{(z, f(z)) : z \in L\} \subset \mathbb{C}^N \times X$. The image of ϕ_t is the disjoint union of $L \times K'$ and the holomorphic graph of f_{1-t} over L , so it is holomorphically convex in $\mathbb{C}^N \times X$. By using [3, Proposition 3.3.2] (a fibred version of the tubular neighbourhood theorem for Stein manifolds) along with the Oka-Grauert principle we can extend ϕ_t to a fibred isotopy of injective holomorphic maps on an open neighbourhood of Z in $\mathbb{C}^N \times X$. Since X has the density property, given $\eta > 0$ there is a fibred holomorphic automorphism $\tilde{\phi}(z, x) = (z, \tilde{\varphi}(z, x))$ of $L \times X$ such that $\text{dist}_X(\tilde{\varphi}(z, f_1(z)), f(0)) < \eta$ for $z \in L$ and $\text{dist}_X(\tilde{\varphi}(z, x), x) < \epsilon/2$ for $z \in L$ and $x \in K'$ (see [10] and [3, Theorems 4.10.5 and 4.12.3]). Note that $f_1 = f$. If $\eta > 0$ is chosen small enough, there exists for each $z \in L$ a unique point $\lambda(z) \in B_\delta \subset \mathbb{C}^n$ such that $\tilde{\Psi}_{\lambda(z)}(\tilde{\varphi}(z, f(z))) = f(0)$. The fibred holomorphic automorphism

$$\phi(z, x) = \left(z, \tilde{\Psi}_{\lambda(z)}^{-1}(\tilde{\varphi}(z, x)) \right), \quad z \in L, x \in X$$

then satisfies the lemma provided $\eta > 0$ is chosen small enough. \square

We will also need the following basic result which we include lacking a reference. (The existence of a Fatou-Bieberbach domain of the first kind containing a point $p \in X$ was proved by Varolin [13], but this is not sufficient for our purpose.)

Lemma 3.3. *Let X be a Stein manifold with the density property, let $K \subset X$ be a holomorphically convex compact set, and let $p \in X \setminus K$. Then there exists a Fatou-Bieberbach domain $\Omega \subset X \setminus K$ of the first kind such that $p \in \Omega$.*

Proof. Let K' be a holomorphically convex compact set in X containing K in its interior and such that $p \notin K'$. Choose local coordinates $\phi : U_p \rightarrow \mathbb{C}^n$ near p such that $\phi(p) = 0$. Let $\delta > 0$ be small enough such that $\phi^{-1}(\overline{\mathbb{B}_\delta(0)}) \cap K' = \emptyset$ and $\phi^{-1}(\overline{\mathbb{B}_\delta(0)}) \cup K'$ is holomorphically convex. Let $F : \overline{\mathbb{B}_\delta(0)} \rightarrow \mathbb{B}_\delta(0)$ be the map $F(z_1, \dots, z_n) = (\frac{z_1}{2}, \dots, \frac{z_n}{2})$. Utilizing the density property of X we now approximate the map $\phi^{-1} \circ F \circ \phi$ on $\phi^{-1}(\overline{\mathbb{B}_\delta(0)})$ and the identity map on K' to obtain a sequence $G_j \in \text{Aut}(X)$ such that for any $k \in \mathbb{N}$ we have that $G_k \circ \dots \circ G_1(K) \subset K'$, and setting $F_j = \phi \circ G_j \circ \phi^{-1}$ we have that

$$s \cdot \|z\| \leq \|F_j(z)\| \leq r \cdot \|z\|, \quad j \in \mathbb{N}$$

on $\mathbb{B}_\delta(0)$, with $r^2 < s < 1$. Now, following [15, proof of Theorem 4] we have that the abstract basin of attraction, or the tail space $\tilde{\Omega}$ (see [1]) associated to $\{F_j\}$, is biholomorphic to \mathbb{C}^n , and the basin of attraction Ω of $\{G_j\}$ is biholomorphic to $\tilde{\Omega}$. \square

Proof of Theorem 3.1: This is an immediate consequence of Lemmas 3.2 and 3.3. \square

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