Diagonally and antidiagonally symmetric alternating sign matrices of odd order

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Abstract

We study the enumeration of diagonally and antidiagonally symmetric alternating sign matrices (DAS-

ASMs) of fixed odd order by introducing a case of the six-vertex model whose configurations are in bijection with

such matrices. The model involves a grid graph on a triangle, with bulk and boundary weights which satisfy the Yang–

Baxter and reflection equations. We obtain a general expression for the partition function of this model as a sum of

two determinantal terms, and show that at a certain point each of these terms reduces to a Schur function. We are then

able to prove a conjecture of Robbins from the mid 1980's that the total number of (2n + 1) × (2n + 1) DASASMs

is $\prod_{i=0}^{n} \frac{(3i)!}{(n+i)!}$, andaconjectureofStroganovfrom2008thattheratiobetweenthenumbersof(2n+1)×(2n+1)

DASASMs with central entry −1 and 1 is n/(n + 1). Among the several product formulae for the enumeration of symmetric alternating sign matrices which were conjectured in the 1980's, that for odd-order DASASMs is the last to have been proved.

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