Chapter 1

The Spin-Charge-Family theory offers the explanation for all the assumptions of the Standard model, for the Dark matter, for the Matter-antimatter asymmetry, making several predictions

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The spin-charge-family theory,\textsuperscript{1}–\textsuperscript{17} which is a kind of the Kaluza-Klein theories but with fermions carrying two kinds of spins (no charges), offers the explanation for all the assumptions of the standard model, with the origin of families, the higgs and the Yukawa couplings included. It offers the explanation also for other phenomena, like the origin of the dark matter and of the matter/antimatter asymmetry in the universe. It predicts the existence of the fourth family to the observed three, as well as several scalar fields with the weak and the hyper charge of the standard model higgs (±\textsuperscript{1}{\textfrac{1}{2}}, ±\textsuperscript{1}{\textfrac{3}{2}}, respectively), which determine the mass matrices of family members, offering an explanation, why the fourth family with the masses above 1 TeV contributes weakly to the gluon-fusion production of the observed higgs and to its decay into two photons,\textsuperscript{18} and predicting that the two photons events, observed at the LHC at ≈750 GeV,\textsuperscript{19–21} might be an indication for the existence of one of several scalars predicted by this theory.

1. Introduction

The spin-charge-family theory\textsuperscript{1}–\textsuperscript{17} offers the explanation for all the assumptions of the standard model: \textbf{i.} For the properties of each family member - quarks and leptons, left and right handed (right handed neutrinos are in this theory regular members of each family) and for anti-fermions. \textbf{ii.} For the appearance of the families. \textbf{iii.} For the existence of the gauge vector fields of the family members charges. \textbf{iv.} For the scalar field and the Yukawa couplings. It is offering the explanation also for the phenomena, which are not included in the standard model, like: \textbf{v.} For the existence of the dark matter,\textsuperscript{13} \textbf{vi.} For the (ordinary) matter-antimatter asymmetry\textsuperscript{2} in the universe.

The spin-charge-family theory predicts that there are at the low energy regime two decoupled groups of four families: The fourth\textsuperscript{3,4,9,10,12} to the already observed three families of quarks and leptons will be measured at the LHC.\textsuperscript{14} At the LHC observed two photons event at ≈750 GeV\textsuperscript{19–21} might be due to the scalar field, which mostly couples to the 4th family quarks.\textsuperscript{22} The lowest of the upper four families builds the dark matter.\textsuperscript{13}
The $4 \times 4$ mass matrices of all the family members demonstrate in this theory the same symmetry,\textsuperscript{14,15} determined by the scalar fields: The two $SU(2)$ triplets - the gauge fields of the two family groups operating among families - and the three singlets - the gauge fields of the three charges, $(Q, Q', Y')$, distinguishing among family members.\textsuperscript{1,2} All these scalar fields carry the weak and the hyper charge as does the scalar of the standard model: $(\pm \frac{1}{2}$ and $\mp \frac{1}{2}$, respectively).\textsuperscript{17}

Since there has been no direct observation of the fourth family quarks with the masses below 1 TeV, while the fourth family quarks with masses above 1 TeV would contribute according to the standard model to either the gluon-fusion scalar field (the higgs) production or to the scalar field decay to two photons $\approx 10$ times too much in comparison with the observations\textsuperscript{a}, the high energy physicists do not expect the existence of the fourth family members at all.\textsuperscript{18}

Might this mean that there does not exist the fourth family coupled to the observed three? Let be pointed out again that the spin-charge-family theory is able - while starting from a very simple action in $d \geq (13 + 1)$, Eqs. (1, 2), with massless fermions with the spin of two kinds (one kind taking care of the spin and the charges of the family members, the second kind taking care of the families\textsuperscript{b}), which couple only to the gravity (through the vielbeins and the two kinds of the corresponding spin connections) - to explain not only all the assumptions of the standard model:

- a. It explains the appearance of all the charges of the left and the right handed members.
- b. It explains the appearance of all the corresponding vector and scalar gauge fields and their properties (explaining the higgs and the Yukawa couplings).
- c. It explains the appearance and properties of family members and their families. It answers also several open questions beyond the standard model, like:
  - d. It explains the appearance and properties of the dark matter.\textsuperscript{13}
  - e. It explains the appearance of the matter/antimatter asymmetry in the universe,\textsuperscript{2} as well as the proton decay.

The more work is done on the spin-charge-family theory, the more explanations of the observed phenomena and the more predictions for the future observations follow.

This paper presents:

- i. In Sect. 2 a short overview of the spin-charge-family theory is made. For more detailed discussions on the properties of the theory the reader is kindly invited to read Refs.,\textsuperscript{1,2} and in these references cited papers. In App. A.1 the technique to represent spinors, used in this talk to explain the properties of the families and the family members, is explained.
- ii. In Sect. 3 the properties of the scalar fields, contributing to the electroweak break, are discussed, showing that all the scalar fields - the three singlets carrying the family members quantum numbers (interacting with both groups of four families) and the twice two triplets carrying the family quantum numbers (each pair of two

\textsuperscript{a}According to the standard model there are the Yukawa couplings which determine the couplings of quarks to the scalar higgs, making them proportional to $m_{\alpha}^2$, if $m_{\alpha}^2$ is the $i^{th}$ family member $(\alpha = u, d)$ mass and $v$ the vacuum expectation value of the scalar.

\textsuperscript{b}The two kinds of spins are connected with the left and the right multiplication of any Clifford algebra object,\textsuperscript{1} respectively.
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tripplets interacts with its own group of four families) - carry with respect to the scalar index the quantum numbers of the weak and the hyper charges \((\pm \frac{1}{2}, \mp \frac{1}{2})\) as does the standard model higgs.

iii. In Sect. 6 couplings of quarks to the scalar fields in the spin-charge-family theory are discussed in order to explain that the appearance of the fourth family might not contradict the observations. All the scalars with the space index \(s = (7, 8)\), Eq. (9, 2) carry the weak and the hyper charge of the standard model higgs. The fourth family quarks couple stronger to the scalar fields which carry the family quantum numbers, while the three observed families couple stronger to the scalar fields carrying the family members quantum numbers what causes a large differences in masses of the first three families.

Couplings to the scalar fields carrying the family quantum numbers \((\vec{\tau}^1, \vec{N}_L)\), Eq. (8) of \(u_4\) appear to contribute to these scalars with the opposite sign than that of \(d_4\), Fig. 1, while the couplings of quarks to the scalars carrying the family members quantum numbers, \((Q, Q', Y')\), are determined besides by the coupling constants of the three singlets also by the eigenvalues of \((Q, Q', Y')\) of the family members \(u_i\) and \(d_i\).

Correspondingly the contribution of the \(u_4\) and \(d_4\) quarks to the production of any superposition of the scalar fields carrying the family quantum numbers in the quark-gluon fusion is weakened, if masses of the scalars are high and these two family members have comparable masses. The fourth family members \(u_4\) and \(d_4\) are expected to have comparable masses\(^{14,15}\) if their masses are determined mostly by the scalar fields which are the superposition of the scalar fields with the family quantum numbers.\(^{22}\)

In this case mostly the top \((u_3)\) quarks, the couplings of which to the superposition of the scalar fields with the family members quantum numbers are strong, contribute to the quark-gluon fusion.

In the case of the weak (but not zero) contribution of the scalar fields with the family members quantum numbers to the \(u_4\) and \(d_4\) mass matrices, a mass eigenstate of the scalar fields which is mostly superposition of scalars with the family quantum numbers can be produced in the quark-gluon fusion. Since such a scalar field mostly couples to the fourth family quarks, and since their matrix elements to the lower three families are very small, the two photons event, observed at the LHC\(^{20,21}\) at \(\approx 750GeV\), might be the first observed signal that there are additional scalars, in agreement with the prediction of the spin-charge-family theory. The strong indirect signal that there are several scalar fields are the existence of the families and the Yukawa couplings of the higgs to the family members of the families.
2. The spin-charge family theory, the starting action and assumptions

I present in this section the assumptions of the spin-charge-family theory, on which the theory is built, following a lot the similar one from Refs.\textsuperscript{1,2}

A i. In the action\textsuperscript{1–4} fermions $\psi$ carry in $d = (13 + 1)$ as the internal degrees of freedom only two kinds of spins (no charges), which are determined by the two kinds of the Clifford algebra objects (there exist no additional Clifford algebra objects (A.1)) - $\gamma^a$ and $\bar{\gamma}^a$ - and interact correspondingly with the two kinds of the spin connection fields - $\omega_{aba}$ and $\bar{\omega}_{aba}$, the gauge fields of $S^{ab} = \frac{1}{4} \left( \gamma^a \gamma^b - \gamma^b \gamma^a \right)$ (the generators of $SO(13,1)$) and $\bar{S}^{ab} = \frac{1}{4} \left( \bar{\gamma}^a \bar{\gamma}^b - \bar{\gamma}^b \bar{\gamma}^a \right)$ (the generators of $\bar{SO}(13,1)$) - and the vielbeins $f_{\alpha}^a$.

$$A = \int d^d x \ E \ L_f + \int d^d x \ E \left( \alpha R + \bar{\alpha} \bar{R} \right),$$

$$L_f = \frac{1}{2} \left( \bar{\psi} \gamma^a p_{\alpha a} \psi \right) + h.c.,$$

$$p_{\alpha a} = f_{\alpha}^a \rho_{\alpha a} \pm \frac{1}{2E} \left\{ p_{\alpha}, E f_{\alpha}^a \right\} - , \quad p_{\alpha a} = p_{\alpha} - \frac{1}{2} S^{ab} \omega_{aba} - \frac{1}{2} \bar{S}^{ab} \bar{\omega}_{aba},$$

$$R = \frac{1}{2} \left\{ f^{\alpha[a} f^{\beta b]} \left( \omega_{aba,\beta} - \omega_{\bar{c}aba} \bar{\omega}^{\beta \bar{c}a} \right) \right\} + h.c.,$$

$$\bar{R} = \frac{1}{2} \left\{ f^{\alpha[a} f^{\beta b]} \left( \bar{\omega}_{aba,\beta} - \bar{\omega}_{\bar{c}aba} \bar{\omega}^{\beta \bar{c}a} \right) \right\} + h.c..$$

Here $c f^{\alpha[a} f^{\beta b]} = f^{\alpha[a} f^{\beta b]} - f^{\rho a} f^{\beta b]}$. $R$ and $\bar{R}$ are the two scalars ($R$ is a curvature).

A ii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ (manifesting as $SO(7,1) \times SU(3) \times U(1)$), affecting both internal degrees of freedom - the one represented by (a superposition of) $S^{ab}$ and the one represented by (a superposition of) $\bar{S}^{ab}$. Since the left handed (with respect to $M^{(7+1)}$) spinors couple differently to scalar (with respect to $M^{(7+1)}$) fields than the right handed ones, the break can leave massless and mass protected $2^{((7+1)/2-1)}$ massless families (which decouple into twice four families). The rest of families get heavy masses $d$.

A iii. The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^{(4)}$.

A iv. The scalar condensate (Table 1) of two right handed neutrinos with the family quantum numbers of one of the two groups of four families, brings masses of

\[ c f^{\alpha a} \] are inverted vielbeins to $e^a_{\alpha}$, with the properties $e^a_{\alpha} f^{\alpha b} = \delta^a_b$, $e^a_{\alpha} f^{\beta a} = \delta^\beta_\alpha$, $E = \det(e^a_{\alpha})$.

Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, .., \sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag(1, -1, -1, .., -1)$.

\[ d \text{ A toy model}^{27,28} \text{ was studied in } d = (5 + 1) \text{ with the same action as in Eq. (1). The break from } d = (5 + 1) \text{ to } d = (3 + 1) \times \text{ an almost } S^2 \text{ was studied. For a particular choice of vielbeins and for a class of spin connection fields the manifold } M^{(5+1)} \text{ breaks into } M^{(3+1)} \text{ times an almost } S^2, \text{ while } 2^{((3+1)/2-1)} \text{ families remain massless and mass protected. Equivalent assumption, its proof is in progress, is made in the case } d = (13 + 1). \]
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Table 1. This table is taken from. The condensate of the two right handed neutrinos $\nu_{IR}$, with the $VIII^{th}$ family quantum numbers (it might be as well that all the neutrinos with the family quantum numbers from (V-VIII), that is from the upper four families, contribute to the condensate), coupled to spin zero ($S^{01} = 0 = S^{12}$) and belonging to a triplet with respect to the generators $\tau_2^3$, while $\tau_1^3 = 0$, is presented, together with its two partners. The right handed neutrino has $Q = 0 = Y$. The triplet carries $\tau^4 = -1$, $\tau^{13} = 0$ $\tau^{23} = 1$, $\tau^4 = -1$, $N^2_R = 1$, $N^3_R = 0$, $Y = 0$, $Q = 0$. The family quantum numbers are presented in Table 5.

<table>
<thead>
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<th>state</th>
<th>$\tau^2$</th>
<th>$\tau^3$</th>
<th>$\tau^4$</th>
<th>$\tau^{13}$</th>
<th>$\tau^{23}$</th>
<th>$\tau^4$</th>
<th>$Y$</th>
<th>$Q$</th>
<th>$N^3_R$</th>
</tr>
</thead>
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<td>$</td>
<td>\nu^I_{II}</td>
<td>_1 &gt;</td>
<td>\nu^I_{II}</td>
<td>_2 &gt; 2$</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>$</td>
<td>\nu^I_{II}</td>
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<td>-1</td>
<td>-2</td>
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</tr>
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the scale of the unification ($> 10^{16}$ GeV) to all the vector and scalar gauge fields, which interact with the condensate.\(^2\)

**A v.** There are nonzero vacuum expectation values of the scalar fields with the space index $s = (7, 8)$, conserving the electromagnetic and colour charge, which cause the electroweak break and bring masses to all the fermions and to the heavy bosons.

Comments on the assumptions:

**C i.** The starting action contains all degrees of freedom, either for fermions or for bosons needed to manifest at low energy regime in $d = (3 + 1)$ all the vector and scalar gauge fields and the one family members as well as families of quarks and leptons as assumed by the standard model: a. One representation of $SO(13, 1)$ contains, if analyzed with respect to the standard model groups ($SO(3, 1) \times SU(2) \times U(1) \times SU(3)$) all the members of one family (Tables 2–4), left and right handed, quarks and leptons (the right handed neutrino is one of the family members), with the quantum numbers required by the standard model\(^c\). b. The action explains the appearance of families due to the two kinds of the infinitesimal generators of groups: $S^{ab}$ and $S^{ab \ d}$. c. The action explains the appearance of the gauge fields of the standard model\(^1,2\) (In Ref.\(^1\) Sect. II. the proof is presented, that gauge fields can in the Kaluza-Klein theories be equivalently represented by either the vielbeins

\(^a\)It contains the left handed weak ($SU(2)_l$) charged and $SU(2)_l$ chargeless colour triplet quarks and colourless leptons (neutrinos and electrons), and the right handed weak chargeless and $SU(2)_l$ charged colour quarks and colourless leptons, as well as the right handed weak charged and $SU(2)_l$ chargeless colour anti-triplet anti-quarks and (anti)colourless anti-leptons, and the left handed weak chargeless and $SU(2)_l$ charged anti-quarks and anti-leptons. The anti-fermion states are reachable from the fermion states by the application of the discrete symmetry operator $C_N \ P_N$, presented in Ref.\(^2\).

\(^b\)There are before the electroweak break two decoupled groups of four massless families of quarks and leptons, in the fundamental representations of $SU(2)_R \times SO(3, 1)$ and $SU(2)_L \times SO(4)$ (Table 5). These eight right families remain massless up to the electroweak break due to the "mass protection mechanism", that is due to the fact that the right handed members have no left handed partners with the same charges.
or spin connection fields.)

b. It explains the appearance of the scalar higgs and Yukawa couplings. The starting action contains also the additional $SU(2)_{II}$ (from $SO(4)$) vector gauge triplet (one of the components contributes to the hyper charge gauge fields as explained above), as well as the scalar fields with the space index $s \in (5, 6)$ and $t \in (9, 10, \ldots, 14)$. All these fields gain masses of the scale of the condensate (Table 1), which they interact with. They all are expressible with the superposition of $f^{m}_s \, \omega_{abs}$ or of $f^{m}_s \, \tilde{\omega}_{abs}$.

c. There are many ways of breaking symmetries from $SU(5)$ vector gauge fields (of the group $SU(2)$ or spin connection fields.)

d. After the break from $SU(5)$ gauge fields are the propagating fields. Observed properties of the family members - the quarks and the leptons, left and right handed (Tables 2–4) - and of the observed vector gauge fields. The starting action contains also the additional $SU(2)_{II}$ to $SO(4)$ to $SU(2) \times U(1) \times SU(3)$ the anti-particles are accessible from particles by the application of the operator $\mathbb{C}_N \cdot \mathbb{P}_N$, as explained in Refs. 26

e. It is the condensate (Table 1) of two right handed neutrinos with the quantum numbers of one group of four families, which makes massive all the scalar gauge fields (with the index $(5, 6, 7, 8)$, as well as those with the index $(9, \ldots, 14)$) and the vector gauge fields, manifesting nonzero $\tau^1, \tau^{23}, \tilde{\tau}^4, \tilde{\tau}^{23}, N^{+}_{R, 1}$. Only the vector gauge fields of $Y$ ($U(1)$), $\tilde{\tau}_3$ ($SU(3)$) and $\tilde{\tau}_1$ ($SU(2)$) remain massless, since they do not interact with the condensate.

f. At the electroweak break the scalar fields with the space index $s = (7, 8)$ - originating in $\tilde{\omega}_{abs}$, as well as some superposition of $\omega_{'s'', s''}$, with the quantum

\[ \tilde{\omega}_{abs} = (13 + 1) \] to $d = (3 + 1)$. The assumed breaks explain the connection between the weak and the hyper charge and the handedness of spinors, manifesting correspondingly the observed properties of the family members - the quarks and the leptons, left and right handed (Tables 2–4) - and of the observed vector gauge fields.

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numbers \((Q, Q', Y')\), conserving the colour and the electromagnetic charge - change their mutual interaction, and gaining nonzero vacuum expectation values change correspondingly also their masses. They contribute to mass matrices of twice the four families, as well as to the masses of the heavy vector bosons.

All the rest scalar fields keep masses of the scale of the condensate and are correspondingly unobservable in the low energy regime.

The fourth family to the observed three ones is predicted to be observed at the LHC. Its properties are under consideration,\textsuperscript{14,15} the baryons of the stable family of the upper four families offer the explanation for the dark matter.\textsuperscript{13} The triplet and anti-triplet scalar fields contribute together with the condensate to the matter/anti-matter asymmetry.

Let us (formally) rewrite that part of the action of Eq.(1), which determines the spinor degrees of freedom, in the way that we can clearly see that the action does in the low energy regime manifest by the standard model required degrees of freedom of the fermions, vector and scalar gauge fields.\textsuperscript{1,3,10–14}

\[
\mathcal{L}_f = \bar{\psi} \gamma^m (p_m - \sum_{A,i} g^A \tau^A \bar{A}_m) \psi + \\
\{ \sum_{s=7,8} \bar{\psi} \gamma^s p_{0s} \psi \} + \\
\{ \sum_{t=5,6,9,\ldots,14} \bar{\psi} \gamma^t p_{0t} \psi \},
\]

where \(p_{0s} = p_s - \frac{1}{2} S^{s'} S_{s' s} - \frac{1}{2} \tilde{S}^{ab} \tilde{S}_{ab s'}\), \(p_{0t} = p_t - \frac{1}{2} S^{t'} S_{t' t} - \frac{1}{2} \tilde{S}^{ab} \tilde{S}_{ab t'}\), with \(m \in (0, 1, 2, 3), s \in (7, 8), (s', s'') \in (5, 6, 7, 8), (a, b)\) (appearing in \(\tilde{S}^{ab}\)) run within either \((0, 1, 2, 3)\) or \((5, 6, 7, 8)\), \(t\) runs either \((5, 6, 7, 8)\) or \((9, 10, \ldots, 14)\). The spinor function \(\psi\) represents all family members of all the \(2^{4+1}=8\) families.

The first line of Eq. (2) determines (in \(d = (3+1)\)) the kinematics and dynamics of spinor (fermion) fields, coupled to the vector gauge fields. The generators \(\tau^{A_i}\) of the charge groups are expressible in terms of \(S^{ab}\) through the complex coefficients \(c^{A_i ab}\).

\[
\tau^{A_i} = \sum_{a,b} c^{A_i ab} S^{ab},
\]

fulfilling the commutation relations

\[
\{\tau^{A_i}, \tau^{B_j}\} = i \delta^{AB} f^{A_ijk} \tau^{Ak}.
\]

The electroweak break the charges \(Y := \tau^4 + \tau^{23}, Y' := -\tau^4 \tan^2 \theta_2 + \tau^{23}, Q := \tau^{13} + Y, Q' := -Y \tan^2 \theta_1 + \tau^{13}\) manifest. \(\theta_1\) is the electroweak angle, breaking \(SU(2)_I\), \(\theta_2\) is the angle of the break of the \(SU(2)_I\) from \(SU(2)_I \times SU(2)_I\).
They represent the colour ($\tau^{3}$), the weak ($\tau^{1}$) and the hyper ($Y$) charges (as well as the $SU(2)_{I}$ ($\tau^{2}$) and $\tau^{4}$ charges, the gauge fields of which gain masses interacting with the condensate, Table 1, leaving massless only the hyper charge vector gauge field). The corresponding vector gauge fields $A^{\mu}$ are expressible with the spin connection fields $\omega_{\mu
u}$, with $(s,t)$ either in $(5,6,7,8)$ or in $(9,\ldots,14)$, in agreement with the assumptions A ii. and A iii.. I demonstrate in Ref.1 the equivalence between the usual Kaluza-Klein procedure leading to the vector gauge fields through the vielbeins and the procedure with the spin connections proposed by the spin-charge-family theory.

All vector gauge fields, appearing in the first line of Eq. (2), except $A^{2\pm}_{m}$ and $A^{Y'}_{m}$ ($= \cos \vartheta_{2} A^{23}_{m} - \sin \vartheta_{2} A^{Y}_{m}$, $Y'$ and $\tau^{4}$ are defined in m), are massless before the electroweak break. $\vec{A}^{1}_{m}$ carries the colour charge $SU(3)$ (originating in $SO(6)$), $\vec{A}^{1}_{m}$ carries the weak charge $SU(2)_{I}$ ($SU(2)_{I}$ and $SU(2)_{II}$ are the subgroups of $SO(4)$) and $A^{Y'}_{m}$ ($= \sin \vartheta_{2} A^{23}_{m} + \cos \vartheta_{2} A^{Y}_{m}$) carries the corresponding $U(1)$ charge ($Y = \tau^{23} + \tau^{4}$). $\tau^{4}$ originates in $SO(6)$ and $\tau^{23}$ is the third component of the second $SU(2)_{II}$ group, $A^{4}_{m}$ and $\vec{A}^{2}_{m}$ are the corresponding vector gauge fields). The fields $A^{2\pm}_{m}$ and $A^{Y'}_{m}$ get masses of the order of the condensate scale through the interaction with the condensate of the two right handed neutrinos with the quantum numbers of one of the group of four families (the assumption A iv., Table 1). (See Ref.1)

Since spinors (fermions) carry besides the family members quantum numbers also the family quantum numbers, determined by $\bar{S}^{ab} = \frac{1}{4}(\gamma^{a}\gamma^{b} - \gamma^{b}\gamma^{a})$, there are correspondingly $2^{(7+1)/2-1} = 8$ families, which split into two groups of families, each manifesting the $SU(2)_{SO(3,1)} \times SU(2)_{SO(4)} \times U(1)$ symmetry.

The eight families of the first member of the eight-plet of quarks from Tables 2–4, for example, that is of the right handed $u_{1R}$ quark, are presented in the left column of Table 5. In the right column of the same table the equivalent eight-plet of the right handed neutrinos $\nu_{1R}$ are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N_{R,L}^{ab}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{N}_{R}$ and $\vec{\tau}$, these families are singlets with respect to $\vec{N}_{L}$ and $\vec{\tau}$ (with the eigenvalues 0). Another group of families contains doublets with respect to $\vec{N}_{L}$ and $\vec{\tau}$, these families are singlets with respect to $\vec{N}_{R}$ and $\vec{\tau}$ (with the eigenvalues 0).

The scalar fields, which are the gauge scalars of $\vec{N}_{R}$ and $\vec{\tau}$, couple only to the four families which are doublets with respect to these two groups. The scalar fields, which are the gauge scalars of $\vec{N}_{L}$ and $\vec{\tau}$, couple only to the four families which are doublets with respect to these last two groups.

If there are no fermions present then the vector gauge fields of the family members charges and of the family charges - $\omega_{abm}$ and $\tilde{\omega}_{abm}$, respectively - are uniquely

$\omega_{\gamma} := -\tau^{4} \tan^{2} \vartheta_{2} + \tau^{23}$, $\tau^{4} = -\frac{1}{4}(S^{0}10 + S^{11}12 + S^{13}14).$
Table 2. The left handed ($\Gamma^{(13,1)} = -1$) (= $\Gamma^{(7,1)} \times \Gamma^{(6)}$) multiplet of spinors - the members of the $SO(13,1)$ group, manifesting the subgroup $SO(7,1)$ - of the colour charged quarks and anti-quarks and the colourless leptons and anti-leptons, is presented in the massless basis using the technique presented in App. A.1. It contains the left handed ($\Gamma^{(3,1)} = -1$) weak charged ($\tau^1 = \pm \frac{1}{2}$) and $SU(2)_{11}$ chargeless ($\tau^3 = 0$) quarks and the right handed weak chargeless and $SU(2)_{11}$ charged ($\tau^3 = \pm \frac{1}{2}$) quarks of three colours ($c^1 = (\tau^3, \tau^3)$) with the “spinor” charge ($\tau^4 = \frac{1}{2}$) and the colourless left handed weak charged leptons and the right handed weak chargeless leptons with the “spinor” charge ($\tau^4 = -\frac{1}{2}$). $S_{12}$ defines the ordinary spin $\pm \frac{1}{2}$. It contains also the states of opposite charges, reachable from particle states by the application of the discrete symmetry operator $C_N$ $P_N$, presented in Refs. Table 2 is separated into three parts.

<table>
<thead>
<tr>
<th>i</th>
<th>(Anti)Octet, $\Gamma^{(7,1)} = (-1) 1$, $\Gamma^{(6)} = (1) -1$ of (Anti)Quarks and (Anti)Leptons</th>
<th>$\Gamma^{(3,1)}$</th>
<th>$\Gamma^{(4)}$</th>
<th>$\tau^{13}$</th>
<th>$\tau^{23}$</th>
<th>$\tau^{33}$</th>
<th>$\tau^{38}$</th>
<th>$\tau^{24}$</th>
<th>Y</th>
<th>Q</th>
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<tr>
<td>1</td>
<td>$u^1_R$</td>
<td>03</td>
<td>12</td>
<td>56</td>
<td>78</td>
<td>9 10</td>
<td>11 12</td>
<td>13 14</td>
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<tr>
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<td>$u^2_R$</td>
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<td>12</td>
<td>56</td>
<td>78</td>
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<td>3</td>
<td>$d^1_R$</td>
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<td>$d^2_R$</td>
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<td>$u^1_L$</td>
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<td>8</td>
<td>$u^2_L$</td>
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<td>10</td>
<td>$u^4_L$</td>
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Table 3. Continuation of Table 2

| i | \( |^{0}v_{i}| \) | \( \psi_{i}^{(3)} \) | \( S_{12}^{(3)} \) | \( \tau_{13}^{(4)} \) | \( \tau_{23}^{(4)} \) | \( \tau_{33}^{(4)} \) | \( \tau_{38}^{(4)} \) | \( Y \) | \( Q \) |
|---|---|---|---|---|---|---|---|---|---|
| 17 \( \nu_{R}^{3} \) | -12 56 78 9 10 11 12 13 14 | 1 1/2 0 1/2 0 -1/2 1/2 1/2 1/2 |
| 18 \( \nu_{R}^{3} \) | -12 56 78 9 10 11 12 13 14 | 1 1/2 0 1/2 0 -1/2 1/2 1/2 1/2 |
| 25 \( \nu_{L} \) | 03 12 56 78 9 10 11 12 13 14 | 1 1/2 0 1/2 0 0 -1/2 0 0 |
| 26 \( \nu_{R} \) | 03 12 56 78 9 10 11 12 13 14 | 1 1/2 0 1/2 0 0 -1/2 0 0 |
| 27 \( \epsilon_{R} \) | 03 12 56 78 9 10 11 12 13 14 | 1 1/2 0 1/2 0 0 -1/2 -1 -1 |
| 28 \( \epsilon_{L} \) | 03 12 56 78 9 10 11 12 13 14 | 1 1/2 0 1/2 0 0 -1/2 -1 -1 |
| 29 \( \epsilon_{L} \) | 03 12 56 78 9 10 11 12 13 14 | -1 1/2 -1 -1/2 0 0 -1/2 -1 -1 |
| 30 \( \nu_{L} \) | 03 12 56 78 9 10 11 12 13 14 | -1 1/2 -1 -1/2 0 0 -1/2 -1 -1 |
| 31 \( \nu_{L} \) | 03 12 56 78 9 10 11 12 13 14 | -1 1/2 -1 -1/2 0 0 -1/2 -1 -1 |
| 32 \( \nu_{L} \) | 03 12 56 78 9 10 11 12 13 14 | -1 1/2 -1 -1/2 0 0 -1/2 -1 -1 |
| $i$ | $|\Psi_i\rangle$ | $\Gamma(0)$ | $\tau^{12}$ | $\tau^{13}$ | $\tau^{23}$ | $\tau^{33}$ | $\tau^{48}$ | $Y$ | $Q$ |
|---|---|---|---|---|---|---|---|---|---|
| 33 | $d_L^1$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 34 | $d_R^1$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 35 | $u_L^1$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 36 | $u_R^1$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 37 | $d_R^2$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 38 | $d_R^3$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 39 | $u_R^2$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 40 | $u_R^3$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 41 | $d_L^2$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 42 | $d_L^3$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 43 | $\tilde{e}_L$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 44 | $\tilde{e}_R$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 45 | $\tilde{\nu}_L$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 46 | $\tilde{\nu}_R$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 47 | $\tilde{\nu}_R$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 48 | $\tilde{\nu}_R$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

Spin-charge-family theory
Table 5. Eight families of the right handed $u_R^1$ (2–4) quark with spin $\frac{1}{2}$, the colour charge ($\tau^3 = 1/2$, $\tau^8 = 1/(2\sqrt{3})$), and of the colourless right handed neutrino $\nu_R$ of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to $(\tilde{N}_L$ and $\tilde{R}^{(1)})$ and a singlet with respect to $(\tilde{N}_R$ and $\tilde{R}^{(2)})$, the other (II) is a singlet with respect to $(\tilde{N}_L$ and $\tilde{R}^{(1)})$ and a doublet with respect to $(\tilde{N}_R$ and $\tilde{R}^{(2)})$. All the families follow from the starting one by the application of the operators $(\tilde{N}_{RL}, \tilde{R}^{(2)})$ (Eq. (A.20)). The generators $(\tilde{N}_{RL}, \tilde{R}^{(2)})$ (Eq. (A.20)) transform $u_R$ to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino $\nu_R$ to all the colourless members of the same family.

<table>
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<tr>
<th>I</th>
<th>$\tilde{u}<em>R^1</em>{R,1}$</th>
<th>$\tilde{u}<em>R^1</em>{R,2}$</th>
<th>$\tilde{u}<em>R^1</em>{R,3}$</th>
<th>$\tilde{u}<em>R^1</em>{R,4}$</th>
<th>$\nu_R2$</th>
<th>$\nu_R3$</th>
<th>$\nu_R4$</th>
<th>$\tilde{\tau}^3$</th>
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<td>$\tilde{u}<em>R^1</em>{R,1}$</td>
<td>$\tilde{u}<em>R^1</em>{R,2}$</td>
<td>$\tilde{u}<em>R^1</em>{R,3}$</td>
<td>$\tilde{u}<em>R^1</em>{R,4}$</td>
<td>$\nu_R2$</td>
<td>$\nu_R3$</td>
<td>$\nu_R4$</td>
<td>$\tilde{\tau}^3$</td>
<td>$\tilde{\tau}^2$</td>
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<tr>
<td>I</td>
<td>$\tilde{u}<em>R^1</em>{R,5}$</td>
<td>$\tilde{u}<em>R^1</em>{R,6}$</td>
<td>$\tilde{u}<em>R^1</em>{R,7}$</td>
<td>$\tilde{u}<em>R^1</em>{R,8}$</td>
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<td>$\nu_R6$</td>
<td>$\nu_R7$</td>
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<th>$\tilde{u}<em>R^1</em>{R,4}$</th>
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<th>$\nu_R3$</th>
<th>$\nu_R4$</th>
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<th>$\tilde{u}<em>R^1</em>{R,3}$</th>
<th>$\tilde{u}<em>R^1</em>{R,4}$</th>
<th>$\nu_R2$</th>
<th>$\nu_R3$</th>
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<th>$\tilde{\tau}^3$</th>
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<th>$\tilde{u}<em>R^1</em>{R,2}$</th>
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<th>$\tilde{u}<em>R^1</em>{R,4}$</th>
<th>$\nu_R2$</th>
<th>$\nu_R3$</th>
<th>$\nu_R4$</th>
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<td>$\tilde{u}<em>R^1</em>{R,6}$</td>
<td>$\tilde{u}<em>R^1</em>{R,7}$</td>
<td>$\tilde{u}<em>R^1</em>{R,8}$</td>
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<td>$\nu_R6$</td>
<td>$\nu_R7$</td>
<td>$\tilde{\tau}^3$</td>
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</table>
Spin-charge-family theory

expressible with the vielbeins.\textsuperscript{1,2}

The scalar fields, the gauge fields with the space index \( s = (7, 8) \), which are either superposition of \( \tilde{\omega}_{ab} \) or of \( \omega_7', t_8 \), determine - after gaining nonzero vacuum expectation values (the assumption A v. and comments C v.) - masses of fermions (belonging to two groups of four families of family members of spinors) and weak bosons.

The condensate (the assumption A iv.), Table 1, gives masses of the order of the scale of its appearance to all the scalar gauge fields, presented in the second and the third line of Eq. (2).

The colour, the weak and the hyper charges (\( \vec{\tau}^1, \vec{\tau}^3, Y \), respectively) of the corresponding gauge fields are before the electroweak break the conserved charges, since the corresponding vector gauge fields don’t interact with the condensate. After the electroweak break, when the scalar fields with the space index \( s = (7, 8) \) - those with the family quantum numbers and those with the quantum numbers \( (Q, Q', Y') \) - start to strongly self interact (Eq. (12)), gaining nonzero vacuum expectation values, the weak charge and the hyper charge are no longer conserved. The only conserved charges are then the colour and the electromagnetic charges.

In Eq. (13) (the reader may have a look at the Ref.,\textsuperscript{1} Eqs. (10, A8, A9)) the scalar fields with the space index \( (7, 8) \), Eq. (8), are presented as superposition of the spin connection fields of both kinds. These scalar fields determine after the electroweak break the mass matrices of the two decoupled groups of four families (Eq. (14)) and of the heavy bosons.

3. The scalar fields contributing to the electroweak break belong to the weak charge doublets

This section follows mainly the equivalent sections in Refs.\textsuperscript{1,2}

It turns out\textsuperscript{2} that all scalars (the gauge fields with the space index \( s \geq 5 \)) of the action (Eq. 1) carry with respect to the space index charges in the fundamental representations: They are either doublets (Table 6), \( s = (5, 6, 7, 8) \), or triplets (Ref.,\textsuperscript{2} Sect. II, Table I), \( s = (9, 10, ..., 13, 14) \). The scalars with the space indices \( s \in (7, 8) \) and \( s \in (5, 6) \) are the \( SU(2) \) doublets (Table 6).

It is demonstrated in this section that all the scalar gauge fields with the space index \( s \in (7, 8) \) carry - with respect to the space index \( s \in (7, 8) \) - the weak and the hyper charge as does the Higgs’s scalar of the standard model \( (\pm \frac{1}{2}, -\frac{1}{2}) \), while they carry in addition either the family quantum numbers, belonging to (one of the two times two \( SU(2) \)) triplets, or the family members quantum numbers, belonging to (one of the three) singlets. These scalars offer the explanation for the origin of the Higgs’s scalar and the Yukawa coupling (of the scalar higgs to fermions) of the standard model.

To see this one must take into account that the infinitesimal generators \( S^{ab} \),

\[
S^{ab} = \frac{i}{4} (\gamma^a \gamma^b - \gamma^b \gamma^a),
\] (5)
determine spins of spinors, while \( \tilde{S}_{ab} \)
\[
\tilde{S}_{ab} = \frac{i}{4} (\tilde{\sigma}^a \gamma^b - \tilde{\sigma}^b \gamma^a),
\]
determine family charges of spinors (Eq. (6)), while \( S^{ab} \) (Assumption A.1), which
apply on the spin connections \( \omega_{bde} (= f^a \_e \_d b) \) and \( \tilde{\omega}_{bde} (= f^a \_e \_d b) \), on either
the space index \( e \) or any of the indices \( b, d, \tilde{b}, \tilde{d} \), operates as follows
\[
S_{ab} A^{d \ldots e \ldots g} = i (\eta^{ae} A^{d \ldots b \ldots g} - \eta^{be} A^{d \ldots a \ldots g}),
\]
in accordance with the Eqs. (A.9, A.10, A.11). The expressions for the infinitesimal
operators of the subgroups of the starting groups (presented in Eq. (3) and the
footnote before this Eq. (3) and determined by the coefficients \( c^{A}_{ai} \) in Eq. (3)) are
the same for all three kinds of degrees of freedom (Eqs. (5, 6, 7)).

All scalars carry correspondingly, besides the quantum numbers determined by
the space index \( s \), also the quantum numbers \( A_i \), Eq. (3), the states of which belong
to the adjoint representations. At the electroweak break all the scalar fields with
the space index \( 7, 8 \), those which belong to one of twice two triplets carrying the
family quantum numbers (\( \tau^A \)) and those which belong to one of the three singlets
carrying the family members quantum numbers \( (Q, Q', Y') \), Eq. (8) start to self
interact, gaining nonzero vacuum expectation values and breaking the weak charge,
the hyper charge and the family charges.

Let me introduce a common notation \( A^{Ai}_s \) for all the scalar fields with \( s = (7, 8) \),
independently of whether they originate in \( \omega_{a^b^s} \) - in this case \( A_i = (Q, Q', Y') \) - or
in \( \tilde{\omega}_{a^b} \) - in this case all the family quantum numbers of all eight families contribute.
\[
A^{Ai}_s \text{ represents } (A^Q_1, A^Q_2, A^Y_\tau, \tilde{A}^\tau_1, \tilde{A}^\tau_2, \tilde{A}^\tau_N) \subseteq A^{Ai}_s,
\]
\( \tau^{Ai} \) represents \( (Q, Q', Y', \tilde{N}_L, \tilde{N}_R) \).

Statement 1: Scalar fields with the space index \( (7, 8) \) carry with respect to this
space index the weak and the hyper charge \( (\mp \frac{1}{2}, \pm \frac{1}{2}) \), respectively.

The proof is presented in Ref. 1. I shall here demonstrate it only briefly.

Let us make a choice of the superposition of the scalar fields so that they are
eigenstates of \( \tau^{13} = \frac{1}{2} (S^{56} - S^{78}) \) (Eq (3) and footnotes at the same page). For
this purpose let us rewrite the second line of Eq. (2) as follows (the momentum \( p_s \)
is left out)

\[
\sum_{s=(7,8), A_i} \bar{\psi} \gamma^s (\tau^{Ai} A^{Ai}_s) \psi =
\]
\[
-\bar{\psi} \left\{ (+) \ (\tau^{Ai} (A^{Ai}_s - i A^{Ai}_s) + (-) (\tau^{Ai} (A^{Ai}_s + i A^{Ai}_s) \right\} \psi,
\]
\[
(\pm) = \frac{1}{2} (\gamma^7 \pm i \gamma^8), \quad A^{Ai}_s := (A^{Ai}_s \mp i A^{Ai}_s),
\]

It is expected that solutions with nonzero momentum lead to higher masses of the fermion fields
in \( d = (3 + 1) \). We correspondingly pay no attention to the momentum \( p_s \), \( s \in (4,8) \), when
having in mind the lowest energy solutions, manifesting at low energies.
Table 6. The two scalar weak doublets, one with \( \tau^{23} = -\frac{1}{2} \) and the other with \( \tau^{23} = +\frac{1}{2} \), both with the "spinor" quantum number \( \tau^4 = 0 \), are presented. In this table all the scalar fields carry besides the quantum numbers determined by the space index also the quantum numbers \( A_i \) from Eq. (8).

<table>
<thead>
<tr>
<th>state</th>
<th>( \tau^{13} )</th>
<th>( \tau^{23} )</th>
<th>spin</th>
<th>( \tau^4 )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A^4_{78} )</td>
<td>( A_7^{4} + iA_8^{4} )</td>
<td>( +\frac{1}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A^4_{56} )</td>
<td>( A_5^{4} + iA_6^{4} )</td>
<td>( -\frac{1}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A^4_{78} )</td>
<td>( A_7^{4} - iA_8^{4} )</td>
<td>( -\frac{1}{2} )</td>
<td>( +\frac{1}{2} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A^4_{56} )</td>
<td>( A_5^{4} - iA_6^{4} )</td>
<td>( +\frac{1}{2} )</td>
<td>( +\frac{1}{2} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

with the summation over \( A_i \) performed, since \( A^4_i \) represent the scalar fields (\( A^Q_i \), \( A^Q_i' \), \( A^Y_i \), \( A^Y_i' \), \( \vec{A}^1_i \), \( \vec{A}^3_i \), \( \vec{A}^2_i \), \( \vec{A}^N_i \), \( \vec{A}^N_i' \)).

The application of the operators \( Y \) \( (Y = \tau^{23} + \tau^4, \tau^{23} = \frac{1}{2}(S^{56} + S^{78}), \tau^4 = -\frac{1}{2}(S^{910} + S^{1112} + S^{1314})) \), \( Q \) \( (Q = \tau^{13} + Y) \) and \( \tau^{13} \) \( (\tau^{13} = \frac{1}{2}(S^{56} - S^{78})) \) on the fields \( (A^4_i + iA^4_i) \) gives \( (S^{56} \text{ is defined in Eq. (7)}) \)

\[
\tau^{13} (A^4_i + iA^4_i) = \pm \frac{1}{2} (A^4_i + iA^4_i), \\
Y (A^4_i + iA^4_i) = \mp \frac{1}{2} (A^4_i + iA^4_i), \\
Q (A^4_i + iA^4_i) = 0. 
\]

Since \( \tau^4, Y, \tau^{13} \) and \( \tau^{1+}, \tau^{1-} \) give zero if applied on \( (A^Q_i, A^{Q_i'} \) and \( A^{Y_i} \) with respect to the indices \( (Q, Q', Y') \), and since \( Y \) and \( \tau^{13} \) commute with the family quantum numbers, one sees that the scalar fields \( A^4_{78} \) \( (=A^Q_7, A^Q_8), A^{Y_7}, A^{Y_8}, \vec{A}^1_7, \vec{A}^3_7, \vec{A}^2_7, \vec{A}^N_7, \vec{A}^N_7' \), rewritten as \( A^4_{78} = (A^4_7 \pm iA^4_8) \), are eigenstates of \( \tau^{13} \) and \( Y \), having the quantum numbers of the **standard model** Higgs' scalar.

These superposition of \( A^4_{78} \) are presented in Table 6 as two doublets with respect to the weak charge \( \tau^{13} \), with the eigenvalue of \( \tau^{23} \) (the second \( SU(2)_{II} \) charge), equal to either \( -\frac{1}{2} \) or \( +\frac{1}{2} \), respectively.

The operators \( \tau^{1+} = \tau^{11} \pm i\tau^{12} \)

\[
\tau^{1+} = \frac{1}{2} [(S^{58} - S^{67}) \pm i(S^{57} + S^{68})],
\]

transform one member of a doublet from Table 6 into another member of the same doublet, keeping \( \tau^{23} \) \( (= \frac{1}{2}(S^{56} + S^{78})) \) unchanged, clarifying the above statement.

After the appearance of the condensate (Table 1), which breaks the \( SU(2)_{II} \) symmetry (bringing masses to all the scalar fields), the weak charge, \( \tau^1 \), and the hyper charge \( Y \) remain the conserved charges \(^9\).

\(^9\)It is \( \tau^{23} \) which determines the hyper charge \( Y (Y = \tau^{23} + \tau^4) \) of these scalar fields, since \( \tau^4 \), if applied on the scalar index of these scalar fields, gives zero, according to Eqs. in the footnote above Eq. (3).
At the electroweak break the scalar fields with the space index \((7,8)\) start to interact among themselves so that the Lagrange density for these gauge fields changes from \(\mathcal{L}_s = E \{ (p_m \Phi_s^{A_i})^\dagger (p_m \Phi_s^{A_i}) - (m_{A_i}'^2) \Phi_s^{A_i} \Phi_s^{A_i}\} \to \)

\[
\mathcal{L}_{sg} = E \sum_{A,i} \{ \left( p_m \Phi_s^{A_i}\right)^\dagger \left( p_m \Phi_s^{A_i}\right) - \left( \lambda^{A_i} + (m_{A_i}^2) \right) \Phi_s^{A_i} \Phi_s^{A_i} \}
\]

\[
+ \sum_{B,j} A^{A_i B_j} \Phi_s \Phi_s^{B_j \dagger} \Phi_s \Phi_s^{B_j \dagger} \}
\]

where \(- \lambda^{A_i} + m_{A_i}^2 = m_{A_i}^2\) and \(m_{A_i}\) manifest as the mass of the \(A^{A_i}_{78}\) scalar.

The operator \(\tau^{1\Xi}_{Ai}(\text{Eq. (11)})\) transforms \(A^{A_i}_{78}_{(2)}\) into \(A^{A_i}_{78}_{(2)} := (A^{A_i}_{78} + i A^{A_i}_{56})\), while \(\tau^{1\Xi}_{Ai}(\text{Eq. (2)})\).

Let me pay attention to the reader, that the term \(\gamma^0_{78}\) \((-)\) \(\tau^{A_i}_{78} A^{A_i}_{78}_{(2)}\) in Eq. (9) transforms the right handed \(u^L_{Ai}\) quark from the first line of Tables 2–4 into the left handed \(u^L_{Ai}\) quark from the seventh line of the same table \(P\), which can, due to the properties of the scalar fields (Eq. (10)), be interpreted also in the standard model way, namely, that \(A^{A_i}_{78}_{(2)}\) ”dress” \(u^L_{Ai}\) giving it the weak and the hyper charge of the left handed \(u^L_{Ai}\) quark, while \(\gamma^0\) changes handedness. Equivalently happens to \(\nu_R\) from the \(25^{th}\) line, which transforms under the action of \(\gamma^0_{78}\) \((-)\) \(\tau^{A_i}_{78} A^{A_i}_{78}_{(2)}\), into \(\nu_L\) from the \(31^{th}\) line. The operator \(\gamma^0_{78}\) \((+)\) \(\tau^{A_i}_{78} A^{A_i}_{78}_{(2)}\) transforms \(d^L_{i}\) from the third line of Tables 2–4 into \(d^L_{i}\) from the fifth line of this table, or \(e_R\) from the \(27^{th}\) line into \(e_L\) from the \(29^{th}\) line, where \(A^{A_i}_{78}_{(2)}\) belong to the scalar fields from Eq. (8).

The term \(\gamma^0_{78}\) \((\mp)\) \(\tau^{A_i}_{78} A^{A_i}_{78}_{(2)}\) of the action (Eqs. (1, 9)) determines the Yukawa couplings. The operator \(\tau^{A_i}\), if representing the first three operators in Eq. (8), (only) multiplies the right handed family member with its eigenvalue. If \(\tau^{A_i}\) represents the last four operators of Eq. (8), the operators \(\gamma^0_{78}\) \((\mp)\) \(\tau^{A_i}_{78} A^{A_i}_{78}_{(2)}\) \((\mp)\) for \((u_R, \nu_R)\) and \((d_R, e_R)\), respectively) transform the right handed family member of one family into the left handed partner of another family within the same group of four families, since these four operators manifest the symmetry twice \((S\bar{U}(2)\bar{S}(3,1) \times S\bar{U}(2)\bar{S}(4))\).

One group of four families carries the family quantum numbers \((\tilde{N}_L, \bar{N}_L)\), the other group of four families carries the family quantum numbers \((\tilde{E}_2, \bar{N}_R)\).

The nonzero vacuum expectation values of the scalar fields of Eq. (8) break the mass protection mechanism of quarks and leptons and determine correspondingly the mass matrices (Eq. (14)) of the two groups of quarks and leptons.

In loop corrections all the scalar and vector gauge fields, which couple to
fermions, contribute. Correspondingly all the off diagonal matrix elements of the mass matrix (Eq. (14)) depend on the family members quantum numbers.

That the scalar fields $A_{\ell t}^{Ai}_{(2)}$ are either triplets as the gauge fields of the family quantum numbers $(\tilde{N}_R, \tilde{N}_L, \tilde{\tau}_2, \tilde{\tau}_1)$; or they are singlets as the gauge fields of $Q = \tau_1^{13} + Y, Q' = -\tan^2 \vartheta_1 Y + \tau_3^{13}$ and $Y' = -\tan^2 \vartheta_2 Y + \tau_2^{13}$, is shown in Ref. 1, Eq. (22).

One finds

$$\tilde{N}_3^{13} A_{\ell L}^{13} = 0,$$

$$Q A_{\ell Q}^{13}_{(2)} = 0,$$

with $Q = S^{56} + \tau_4 = S^{56} - \frac{1}{2}(S^{10} + S^{11} + S^{12})$, and with $\tau_4$ defined in the footnote on the page of Eq. (3), if replacing $S^{ab}$ by $S^{ab}$ from Eq. (7).

Similarly one finds properties with respect to the $\ell t$ quantum numbers for all the scalar fields $A_{\ell t}^{Ai}_{s,s}$. All other scalar fields: $A_{\ell t}^{Ai}_{s,s}, s \in (5, 6)$ and $A_{\ell t}^{Ai}_{t,t'}, (t, t') \in (9, \ldots, 14)$ have masses of the order of the condensate scale and contribute to matter-antimatter asymmetry.

4. Mass matrices and properties of quarks

One of the most important open questions in the elementary particle physics is: Where do the families originate? Explaining the origin of families would give us the answer about the number of families which will be observed at the low energy regime, about the origin of the scalar field(s) and the Yukawa couplings, telling how many scalar fields can we expect to observe at the acceptable energies and would also explain differences in the fermions properties - the differences in masses and mixing matrices among family members – quarks and leptons, as well as it might explain the hierarchy problem.

The spin-charge-family theory predicts that there are at the low energy regime twice four families of quarks and leptons. This means that besides the observed three there is the fourth family of quarks and leptons. It also mean that the stable of the upper four families must also be observed. I shall comment the agreement of the existence of the fourth family quarks with the observations, in particular with the contribution of the fourth family to the production of the higgs in the quark-fusion process, in Sect. 6. The properties of the lowest of the upper four families, contributing to the dark matter, will be shortly presented in Sect. 7.13

The mass matrix of any of the family member (quark or lepton) demonstrates in the massless basis the $U(1) \times SU(2) \times SU(2)$ symmetry (each of the two $SU(2)$ is a subgroup, one of $SO(3, 1)$ and the other of $SO(4)$) symmetry, Eq. (14).

To the masses of the lower and upper four families all the scalars with the family members quantum numbers $(Q, Q', Y')$ contribute, while the scalars with the family quantum numbers split the eight families into twice four families: To the masses
of the lower four families the scalar fields, which are the gauge fields of $\tilde{N}_L$ and $\tilde{F}_1^3$ contribute. To the masses of the upper four families the gauge fields of $\tilde{N}_R$ and $\tilde{F}_2^2$ contribute.

$$M_\alpha = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}.$$

(14)

Although any accurate $3 \times 3$ submatrix of the $4 \times 4$ unitary matrix determines the $4 \times 4$ matrix uniquely, neither the quark nor (in particular) the lepton $3 \times 3$ mixing matrix are measured accurately enough that it would be possible to determine three complex phases of the $4 \times 4$ mixing matrix as well as the mixing matrix elements of the fourth family members to the lower three. We therefore assumed in our calculations\textsuperscript{12,14,15} that the mass matrices are symmetric and real. Correspondingly the mixing matrices are orthogonal. We fitted the 6 free parameters of each quark mass matrix, Eq. (14), to twice three measured quark masses (6), and to the 6 (from the experimental data extracted) parameters of the corresponding $4 \times 4$ mixing matrix.

While the experimental accuracy of the quark masses of the lower three families does not influence the calculated mass matrices considerably, it turned out that the experimental accuracy of the $3 \times 3$ quark mixing matrix is not good enough to trustworthily determine the mass intervals for the fourth family quarks. Taking into account our calculations fitting the experimental data (and the meson decays evaluations in literature, as well as our own) we estimated that the fourth family quarks masses might be above 1 TeV. Choosing the masses of the fourth family quarks we were able not only to calculate the fourth family matrix elements to the lower three families, but also predict towards which values will the matrix elements of the $3 \times 3$ submatrix move in the more accurate experiments.\textsuperscript{15}

The higher are the fourth family quark masses the closer are the mass matrices to the democratic ones. The fourth family quark masses are closer to each other, the smaller is the contribution of the scalar fields with the family members quantum numbers to the fourth family masses.

The complex mass matrices would lead to unitary and not to orthogonal mixing matrices. The more accurate experimental data for quarks mixing matrix would allow us to extract also the phases of the unitary mixing matrix, allowing us to predict the fourth family masses.

5. Triplets with respect to space index $s = (9, \ldots, 14)$ and the matter-antimatter asymmetry in the universe

The gauge fields with the space index $t \in (9, \ldots, 14)$ form the triplets and antitriplets with respect to the space index $s = (9, \ldots, 14)$ (one triplet and one antitriplet for each $Ai$). I kindly ask the reader to study this topic in Ref.\textsuperscript{2}
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The colour triplet scalars namely contribute to transition from antileptons into quarks and antiquarks into quarks and back, unless the scalar condensate of the two right handed neutrinos, presented in Table 1, breaks matter-antimatter symmetry, offering the explanation for the matter-antimatter asymmetry in our universe. The colour triplet scalars and the condensate cause also the decay of proton.

The condensate breaks also the $SU(2)_{II}$ symmetry, leaving massless besides gravity only the colour, weak and the hyper charge vector gauge fields, the corresponding charges remain conserved. Also all scalar fields get masses through the interaction with the condensate.

There are no additional scalar indices and therefore no additional corresponding scalars with respect to the scalar indices in this theory. Scalars, which do not get nonzero vacuum expectation values, keep masses on the condensate scale.

6. The fourth family quarks and their couplings to the scalar fields

The spin-charge-family theory predicts the fourth family to the observed three, while there has been no direct observation of the fourth family quarks with the masses below 1 TeV. The fourth family quarks with masses above 1 TeV contribute according to the standard model (the standard model Yukawa couplings of quarks to the scalar higgs is proportional to $m_{\alpha}^4 v$, where $m_{\alpha}$ is the fourth family member ($\alpha = u, d$) mass and $v$ the vacuum expectation value of the scalar) to either the quark-gluon fusion production of the scalar field (the higgs) or to the scalar field decay into two photons $\approx 10$ times too much in comparison with the observations. Correspondingly the high energy physicists do not expect the existence of the fourth family members at all.

I am stressing\(^2\) in this section that the $u_i$-quarks and $d_i$-quarks of all the families if coupled with the opposite sign to the scalar fields, carrying the family quantum numbers, $A_i^{\pm}(\tilde{A}_i = (\tilde{\tau}_1^i, \tilde{N}_L^i))$, (Eq. 8)) (they are the same for all the family members) do not contribute to either the quark-gluon fusion production of the scalar fields with the family quantum numbers or to the decay of these scalars into two photons, if the $u_i$-quarks and $d_i$-quarks have the same mass. Since the $u_4$-quarks and $d_4$-quarks might have similar masses, if their masses are mostly determined by the scalars with the family quantum numbers, the observations are consequently not in contradiction with the spin-charge-family theory prediction that there exists the fourth family coupled to the observed three.

The couplings of $u_i$ and $d_i$ to the scalars carrying the family members quantum numbers are determined besides by the corresponding couplings by also by the eigenvalues of the operators ($Q, Q', Y'$) on the quarks states. The strong influence of the scalar fields carrying the family members quantum numbers on the masses of the lower (observed) three families manifests in the huge differences in the masses of $u_i$ and $d_i$, $i = (1, 2, 3)$, among families ($i$) and family members ($a, d$). For the fourth family quarks, which are more and more decoupled from the observed three families the higher are their masses,\(^{14,15}\) the influence of the scalar fields carrying the
family members quantum numbers on their masses is expected to be much weaker. Correspondingly the \( u_4 \) and \( d_4 \) masses become closer to each other the higher are their masses and the weaker is their couplings (the mixing matrix elements) to the lower three families.

If the masses of the fourth family quarks are close to each other, then \( u_4 \) and \( d_4 \) contribute in the quark-gluon fusion very little to the production of the scalar field - the higgs - which is mostly superposition of the scalar fields with the family members quantum numbers, what is in agreement with the observation: In the quark-gluon fusion production of the higgs mostly the top (\( u_3 \)) contributes.

The \( u_4 \) and \( d_4 \) quarks of almost the same mass couple weakly to scalars which carry the family members quantum numbers and correspondingly contribute weakly to the production.

The \( u_4 \) and \( d_4 \) can still, but only weakly, contribute to the production of some of the remaining superposition of the scalar fields, predicted by the spin-charge-family theory, most probably to one of those, which is mainly the superposition of scalar fields carrying the family quantum numbers. This might be the scalar with the mass of \( \approx 750 \text{ GeV} \), observed at the LHC, if this is a real event.

Let me write the statement.\(^{22}\)

**Statement 2:** The \( u_i \)-quarks and \( d_i \)-quarks of equal masses, which couple with the opposite sign to the scalar fields, carrying the family quantum numbers \((\tilde{\Phi}^i_A, \tilde{A} = (\tilde{T}_i, \tilde{N}_L)\), (Eq. 8)), can not contribute in the quark-gluon fusion to the production of these scalar fields.

Let me briefly comment **Statement 2**, making use of the technique,\(^{23,24}\) discussed in App. A.1.

From Tables 2–4 we can see presentation of spinors in terms of projectors and nilpotents, and read their properties. We need here their weak and hyper charges, while taking into account that antiquarks, Tables 2–4, carry the opposite charges than the corresponding quarks and that one can obtain the antiquarks as well by the application of the operator\(^{20,21}\) \( C_N P_N' = \gamma^0 \prod_{n=0}^d \gamma^n \gamma^i I_{x^0, x^1, x^2, \ldots, x^d} \) on the corresponding quarks\(^8\). In Tables 2–4 the phases of all the states are chosen to be 1. In this talk I use different phases, those presented in footnote \(^i\), which enable the usual presentation of fermions under the change of spin and under \( C_N \cdot P_N \).

In Table 7, the properties of \( u \) and \( d \) (and their antiquarks) needed in Fig. 1, are presented.

In Fig. 1 the properties of the \( u \) and \( d \) quarks, contributing to the production of the dynamical part of the scalar fields - \( \Phi_{78}^{A_i} \), Eq.(8), \( (A_{78}^{A_i} = \Phi_{78}^{A_i} + v_{78}^{A_i}) \) - in the

\[^{9}\text{Here } \gamma^0 \text{ and } \gamma_1 \text{ are real, } \gamma^2 \text{ imaginary, } \gamma^3 \text{ real, } \gamma^5 \text{ imaginary, } \gamma^6 \text{ real, alternating imaginary and real up to } \gamma^d, \text{ which is in even dimensional spaces real. } \gamma^n \text{’s appear in the ascending order. Operators } I_n \text{ operate as follows: } I_{-n} x^0 = -x^0; I_{-n} x_a = -x^a; I_{-n} x_a = (-x^0, \tilde{x}) \text{; } I_{-n} x^0 = (x^0, -x^1, -x^2, -x^3, x^4, \ldots, x^d); I_{-n} x_a = x^a; I_{-n} x_a = x^a; I_{-n} x_a = x^a; \text{ for } n = 0, 1, \ldots, d \text{.}
\]
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Table 7. The weak, hyper and elm charges for quarks and antiquarks in their massless basis are presented, the colour charge is not shown. These and other properties of quarks and antiquarks, leptons and antileptons can be read from Tables 2–4.

<table>
<thead>
<tr>
<th>state</th>
<th>τ^{13}</th>
<th>Y</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_{Ri}</td>
<td>\frac{1}{2}</td>
<td>\frac{1}{2}</td>
<td>\frac{1}{2}</td>
</tr>
<tr>
<td>u_{Li}</td>
<td>\frac{1}{2}</td>
<td>\frac{1}{2}</td>
<td>\frac{1}{2}</td>
</tr>
<tr>
<td>d_{Ri}</td>
<td>0</td>
<td>\frac{1}{2}</td>
<td>\frac{1}{2}</td>
</tr>
<tr>
<td>d_{Li}</td>
<td>\frac{1}{2}</td>
<td>\frac{1}{2}</td>
<td>\frac{1}{2}</td>
</tr>
</tbody>
</table>

quark-gluon fusion, are presented. One notices the opposite signs of the couplings of u_i with respect to d_i for ether \Phi^- or for \Phi^+.

Let us discuss the possibility that at the LHC\textsuperscript{20,21} observed two photons effect, if trustworthy, might have the origin in one of the nine mass eigenstates of the scalar fields, contributing to mass matrices (Eq. (14)) of the lower group of four families. Namely, if the masses of u_4 and d_4 are not completely equal (while the couplings of these four family quarks contribute with the opposite signs to the production of scalars with the family quantum numbers as presented in Fig. 1), then besides the observed scalar field (the higgs) also another mass eigenstate, the superposition of mostly the scalar fields with the family quantum numbers, can be produced. Such a scalar decays mostly through the production of two photons, since the mixing matrix elements of the fourth family quarks to the lower (observed) three families are very small.\textsuperscript{14,15} If the event, reported in Refs.,\textsuperscript{20,21} is a true one, this scalar field would have a mass of \approx 750 GeV.

The production of a photon by quarks is presented in Figs. 2 (a, b) for u_L and d_L with spin up, as an example. Only the weak (\tau^{13}) and hyper (Y) charges besides the spin (S^{12}, presented in figures by arrows) are shown. In this case the phases are, as expected, the same for u and d, up to the sign determined by the electromagnetic charge, which are for u and d different. Equivalent figures can be drawn for gluons, where the colour charge replaces the electromagnetic charge of quarks.\textsuperscript{22} The figures are valid for any A_i and correspondingly also for any superposition of \Phi_{A_i}.

7. The upper groups of four families and the dark matter

As discussed in Sect. 2 the spin-charge-family theory\textsuperscript{1–17} predicts in the low energy region two decoupled groups of four families. In Ref.\textsuperscript{13} the possibility that the dark matter consists of clusters of the fifth family - the stable heavy family of quarks and leptons (with zero Yukawa couplings to the lower group of four families) - is discussed.

We made in Ref.\textsuperscript{13} a rough estimation of the properties of baryons of this fifth family members, of their behaviour during the evolution of the universe and when scattering on the ordinary matter. We studied possible limitations on the family properties due to the cosmological evidences, the direct experimental evidences, and
operators $\tilde{u}$ and $u$ family members when $\tau_{Q,Q}$ do not distinguish among family members $u$ and $d$ so that in this case the contribution of $u$ and $d$ have opposite signs, $Q, Q', Y'$ do, influencing the signs in addition.

all others (at that time) known properties of the dark matter.

We used the simple hydrogen-like model to evaluate the properties of these heavy baryons and their interaction among themselves and with the ordinary nuclei, taking into account that for masses of the order of $1 \text{ TeV}$ or larger the one gluon exchange determines the force among the constituents of the fifth family baryons. Due to their very large masses "the nuclear interaction" among these baryons has very interesting properties. We concluded that it is the fifth family neutron, which is very probably the most stable nucleon.

We followed the behaviour of the fifth family quarks and antiquarks in the plasma of the expanding universe, through the freezing out procedure, through the colour phase transition, while forming neutrons, up to the present dark matter. Also the scattering of the fifth family neutrons among themselves and on the ordinary matter...
Fig. 2. The contribution of quarks to the production of a photon is presented, to manifest the difference in the production of the scalar fields and photons.

was evaluated.

The cosmological evolution suggested the limits for the masses of the fifth family quarks

$$10 \text{ TeV} < m_{q_5} c^2 < \text{a few } \cdot 10^2 \text{ TeV} \quad (15)$$

and for the scattering cross sections

$$10^{-8} \text{ fm}^2 < \sigma_{cs} < 10^{-6} \text{ fm}^2, \quad (16)$$

while the measured density of the dark matter does not put much limitation on the properties of heavy enough clusters.

The direct measurements limited the fifth family quark mass to \(^{13}\)

several \(10 \text{ TeV} < m_{q_5} c^2 < 10^5 \text{ TeV} \).

We also find that our fifth family baryons of the mass of a few hundreds TeV/c^2 have for a factor more than 100 times too small scattering amplitude with the ordinary matter to cause a measurable heat flux on the Earth’s surface.

8. Concluding remarks

One of the most important open questions in the elementary particle physics is: Where do the families originate? Explaining the origin of families would answer the question about the number of families which are possibly observable at the low energy regime, about the origin of the scalar field(s) and the Yukawa couplings (the couplings of fermions to the scalar field(s)), about the differences in the fermions properties - the differences in the masses and mixing matrices among family members – quarks and leptons, as well as about the hierarchy in quark and lepton masses.

To understand the history of the universe is needed to explain the assumptions of the standard model, as well as the phenomena like the existence of the dark matter, dark energy and matter/antimatter asymmetry.
I demonstrated in this talk, that the spin-charge-family theory, starting with the simple action in $d = (13 + 1)$ for fermions which carry two kinds of spins (no charges) and couple correspondingly to the vielbeins and the two kinds of spin connection fields and the corresponding boson fields, offers the explanation for all the assumptions of the standard model:

a. The theory explains all the properties of the family members - quarks and leptons, left and right handed, and their right and left handed antiquarks and antileptons, explaining why the left handed spinors carry the weak charge while the right handed do not (the right handed neutrino is the regular member of each family).

b. It explains the appearance and the properties of the families of family members.

c. It explains the existence of the gauge vector fields of the family members charges.

d. It explains the appearance and the properties of the scalar field (the higgs) and the Yukawa couplings.

It also offers the explanation for the phenomena, which are not integrated into the standard model, like:

e. It explains the existence of the dark matter.

f. It explain the origin of the (ordinary) matter/antimatter asymmetry in the universe.

The theory predicts:

g. There are twice two groups of four families of quarks and leptons at low energies.

g.i. The fourth family with masses above 1 TeV, weakly coupled to the observed three families, will be measured at the LHC.

g.ii. The quarks and leptons of the fifth family - that is of the stable one of the upper four families - form the dark matter. The family members, which form the chargeless clusters, manifest, due to their very heavy masses, a ”new nuclear force”.

h. The predicted scalar fields with the space index $(7, 8)$ are doublets with respect to the space index (carrying the weak and the hyper charge of the standard model higgs). They carry in addition:

h.i. Either they carry one of the three family members quantum numbers, $(Q, Q', Y')$ - belonging correspondingly to one of three singlets.

h.ii. Or they carry family quantum numbers - belonging correspondingly to one of the twice two triplets.

\[\text{If there are no spinors present, are the two spin connections expressible uniquely with the vielbeins.}\]

\[\text{One Weyl representation of } SO(13 + 1) \text{ contains, if analyzed with respect to the standard model groups, all the members of one family, the coloured quarks and colourless leptons, and the anticoloured antiquarks and (anti)colourless antileptons, with the left handed leptons carrying the weak charge and the right handed ones weak chargeless, while the left handed antispinors are weak chargeless and the right handed ones carry the weak charge.}\]
h.iii. The three singlets and the two triplets determine mass matrices of the lower four families, contributing to masses of the heavy vector bosons.

h.iv. These scalars determine the observed higgs and the Yukawa couplings.

i. The predicted scalar fields with the space index \((9, 10, ..., 14)\) are triplets with respect to the space index. They cause the transitions from antileptons into quarks and antiquarks into quarks and back. The condensate breaks the matter/antimatter symmetry, causing the asymmetry in the (ordinary) matter with respect to antimatter.

i.i. The condensate is responsible also for the proton decay.

j. The condensate is a scalar of the two right handed neutrinos with the family quantum numbers of the upper four families.

k. The condensate gives masses to all the gauge fields with which it interacts.

k.i. It gives masses to all scalar fields and to vector fields, leaving masses only the colour, the weak, the hyper vector gauge fields and the gravity in \(\text{SU}(3 + 1)\).

l. There is the \(\text{SU}(2)\) (belonging together with the weak \(\text{SU}(2)\) to \(\text{SO}(7, 1)\)) vector gauge field, which gain masses of the order of the appearance of the condensate.

m. At the electroweak break the scalar fields with the space index \((7, 8)\) change their mutual interaction, and gaining nonzero vacuum expectation values, break the weak and the hyper charges and correspondingly the mass protection of fermions, making them massive.

n. The symmetry of mass matrices allow, in the case that the experimental data for the mixing submatrix \(3 \times 3\) of the \(4 \times 4\) mixing matrix are accurately enough, to determine the mixing matrix and the masses of the fourth family quarks. The accuracy, with which the masses of the six lower families are measured so far, does not influence the results appreciably. Due to uncertainty of the experimental data for the \(3 \times 3\) mixing submatrix we are only able to determine the \(4 \times 4\) quark mixing matrix for a chosen masses of the the fourth family quarks. However, we also predict how will the \(3 \times 3\) submatrix of the mixing matrix change with more accurate measurements.

n.i. The fourth family quarks mass matrices are for masses above 1 TeV closer and closer to the democratic matrices. The less the scalars with the family members quantum numbers contribute to masses of the fourth family quarks, the closer is \(m_{u_4}\) to \(m_{d_4}\).

n.ii. The large contribution of the scalars with the family members quantum numbers \((Q, Q', Y')\) to the masses of the lower four families manifests in the large differences of quarks masses of the lower four families.

n.iii. Although we have done calculations also for leptons, must further analyses of their properties wait for more accurate experimental data.

o. In the case that the \(u_4\) and \(d_4\) quarks have similar masses - determined mostly by the scalar fields carrying the family quantum numbers - they contribute mostly to the production of these scalars, while their contribution to the production of
those scalars which carry the family members quantum numbers - to the higgs in particular - is much weaker, which is in agreement with the experiment.\footnote{The coupling constants of the singlet scalar fields differ among themselves and also from the coupling constants of the two triplet scalar fields}

\textbf{o.i.} The fourth family quarks might contribute to the production of the scalar field, which is a superposition of mostly the scalars carrying the family quantum numbers. The event, observed at the LHC as the two photon production, might be the signal of this new scalar field with the mass of $\approx 750$ GeV.

\textbf{p.} All the degrees of freedom discussed in this talk are already a part of the simple starting action Eq.(1).

\textbf{p.i.} The way of breaking symmetries (ordered by the conditions determining the history of our universe) is assumed so that it leads in $d = (3 + 1)$ to the observable symmetries.

\textbf{p.ii.} Also the effective interaction among scalar fields is assumed, although we could derive it in principle from the starting action.

There are several open problem in the \textit{spin-charge-family} theory:

\textbf{r.} Since this theory is, except that fermions carry two kinds of spins - one kind taking care of spin and charges, the second one taking care of families - a kind of the Kaluza-Klein theories, it shares at very high energy with these theories the quantization problem.

\textbf{s.} The dimension of space-time, $d = (13 + 1)$, is in the \textit{spin-charge-family} theory chosen, since $SO(13,1)$ contains all the members, assumed in the \textit{standard model}. It contains also the right handed neutrino (which carries the $Y'$ quantum number.)

\textbf{s.i.} It must be shown, however, how has nature ”made the decision” in evolution to go through this dimension and what is indeed the dimension of space-time (infinite?)

\textbf{t.} There are many other open question, like:

\textbf{t.i.} What is the reason for the (so small) dark energy?

\textbf{t.ii.} At what energy the electroweak phase transition occurs?

\textbf{t.iii.} Why do we have fermions and bosons?\footnote{31}

\textbf{t.iv.} Some of them might be solved by comparison with other approaches and theories.

\section*{Appendix A.}

\textbf{A.1. Short presentation of spinor technique}\footnote{1,7,23,24} \footnote{1} This appendix is a short review (taken from\footnote{3}) of the technique,\footnote{7,23–25} initiated and developed in Ref.\footnote{7}, while proposing the \textit{spin-charge-family} theory.\footnote{1–17} All the internal degrees of freedom of spinors, with family quantum numbers included, are describable in the space of $d$-anticommuting (Grassmann) coordinates,\footnote{7} if the dimension of ordinary space is also $d$. There are two kinds of operators in the
Grassmann space fulfilling the Clifford algebra and anticommuting with one another Eq.(A.1). The technique was further developed in the present shape together with H.B. Nielsen.\textsuperscript{23–25}

In this last stage we rewrite a spinor basis, written in Ref.\textsuperscript{7} as products of polynomials of Grassmann coordinates of odd and even Grassmann character, chosen to be eigenstates of the Cartan subalgebra defined by the two kinds of the Clifford algebra objects, as products of nilpotents and projections, formed as odd and even objects of \( \gamma^a \)'s, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by \( \gamma^a \)'s and \( \tilde{\gamma}^a \)'s.

The technique can be used to construct a spinor basis for any dimension \( d \) and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum properties of the states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

Ref.\textsuperscript{1} App. B, briefly represents the starting point\textsuperscript{7} of this technique. There are two kinds of the Clifford algebra objects, \( \gamma^a \)'s and \( \tilde{\gamma}^a \)'s.

These objects have properties,

\[
\{ \gamma^a, \gamma^b \} = 2\eta^{ab}, \quad \{ \tilde{\gamma}^a, \tilde{\gamma}^b \} = 2\eta^{ab}, \quad \{ \gamma^a, \tilde{\gamma}^b \} = 0. \tag{A.1}
\]

If \( B \) is a Clifford algebra object, let say a polynomial of \( \gamma^a \), \( B = a_0 + a_a \gamma^a + a_{ab} \gamma^a \gamma^b + \cdots + a_{a_1 a_2 \cdots a_d} \gamma^{a_1} \gamma^{a_2} \cdots \gamma^{a_d} \), one finds

\[
(\tilde{\gamma}^a B : = i(-)^{a_b} B \gamma^a) |\psi_0 \rangle >, \\
B = a_0 + a_a \gamma^a + a_{a_1 a_2} \gamma^{a_1} \gamma^{a_2} + \cdots + a_{a_1 \cdots a_d} \gamma^{a_1} \cdots \gamma^{a_d}, \tag{A.2}
\]

where \( |\psi_0 \rangle > \) is a vacuum state, defined in Eq. (A.16) and \((-)^{a_b} \) is equal to 1 for the term in the polynomial which has an even number of \( \gamma^b \)'s, and to \(-1 \) for the term with an odd number of \( \gamma^b \)'s, for any \( d \), even or odd, and \( I \) is the unit element in the Clifford algebra.

It follows from Eq. (A.2) that the two kinds of the Clifford algebra objects are connected with the left and the right multiplication of any Clifford algebra objects \( B \) (Eq. (A.2)).

The Clifford algebra objects \( S^{ab} \) and \( \tilde{S}^{ab} \) close the algebra of the Lorentz group

\[
\begin{align*}
S^{ab} : = (i/4)(\gamma^a \gamma^b - \gamma^b \gamma^a), \\
\tilde{S}^{ab} : = (i/4)(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a),
\end{align*} \tag{A.3}
\]

\[
\{ S^{ab}, \tilde{S}^{cd} \} = 0, \quad \{ S^{ab}, S^{cd} \} = i(\eta^{ad} S^{bc} + \eta^{bd} S^{ac} - \eta^{ac} S^{bd} - \eta^{bc} S^{ad}), \quad \{ \tilde{S}^{ab}, \tilde{S}^{cd} \} = i(\eta^{ad} \tilde{S}^{bc} + \eta^{bd} \tilde{S}^{ac} - \eta^{ac} \tilde{S}^{bd} - \eta^{bc} \tilde{S}^{ad}).
\]

We assume the “Hermiticity” property for \( \gamma^a \)'s

\[
\gamma^a \dagger = \eta^{aa} \gamma^a, \tag{A.4}
\]

in order that \( \gamma^a \) are compatible with (A.1) and formally unitary, i.e. \( \gamma^a \dagger \gamma^a = I \).

One finds from Eq. (A.4) that \( (S^{ab})^\dagger = \eta^{aa} \eta^{bb} S^{ab} \).
Recognizing from Eq.(A.3) that the two Clifford algebra objects $S^{ab}, S^{cd}$ with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

\begin{align*}
S^{03}, S^{12}, \ldots, S^{d-1 \, d}, & \text{ if } d = 2n \geq 4, \\
S^{03}, S^{12}, \ldots, S^{d-2 \, d-1}, & \text{ if } d = (2n + 1) > 4,
\end{align*}

\begin{align*}
\tilde{S}^{03}, \tilde{S}^{12}, \ldots, \tilde{S}^{d-1 \, d}, & \text{ if } d = 2n \geq 4, \\
\tilde{S}^{03}, \tilde{S}^{12}, \ldots, \tilde{S}^{d-2 \, d-1}, & \text{ if } d = (2n + 1) > 4. \quad (A.5)
\end{align*}

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness $\Gamma (\{\Gamma, S^{ab}\} = 0)$ in any $d$

\begin{align*}
\Gamma^{(d)} : = (i)^{d/2} \prod_a (\sqrt{\gamma^{aa}}), & \text{ if } d = 2n, \\
\Gamma^{(d)} : = (i)^{(d-1)/2} \prod_a (\sqrt{\gamma^{aa}}), & \text{ if } d = 2n + 1. \quad (A.6)
\end{align*}

One proceeds equivalently for $\bar{\Gamma}^{(d)}$, substituting $\gamma^{a}$'s by $\tilde{\gamma}^{a}$'s. We understand the product of $\gamma^{a}$'s in the ascending order with respect to the index $a$: $\gamma^{0} \gamma^{1} \ldots \gamma^{d}$. It follows from Eq.(A.4) for any choice of the signature $\eta^{aa}$ that $\Gamma^{1} = \Gamma$, $\Gamma^{2} = I$. We also find that for $d$ even the handedness anticommutes with the Clifford algebra objects $\gamma^{a} (\{\gamma^{a}, \Gamma\} + = 0)$, while for $d$ odd it commutes with $\gamma^{a} (\{\gamma^{a}, \Gamma\} - = 0)$.

To make the technique simple we introduce the graphic presentation as follows

\begin{align*}
\gamma^{ab} (k) : = \frac{1}{2} (\gamma^{a} + \frac{\eta^{aa}}{ik} \gamma^{b}), & \quad [k] : = \frac{1}{2} (1 + i \frac{1}{k} \gamma^{a} \gamma^{b}), \quad (A.7)
\end{align*}

where $k^2 = \eta^{aa} \eta^{bb}$. It follows then

\begin{align*}
\gamma^{a} = \gamma^{ab} (k) + (-\gamma^{ab} (-k)), & \quad \gamma^{b} = i k \eta^{aa} \gamma^{ab} (k) - (-\gamma^{ab} (-k)), \\
S^{ab} = \frac{k}{2} ([k] - [-k]). \quad (A.8)
\end{align*}

One can easily check by taking into account the Clifford algebra relation (Eq. (A.1)) and the definition of $S^{ab}$ and $\tilde{S}^{ab}$ (Eq. (A.3)) that the nilpotent $(k)$ and the projector $[k]$ are "eigenstates" of $S^{ab}$ and $\tilde{S}^{ab}$

\begin{align*}
S^{ab} (k) = \frac{1}{2} k \; [k], & \quad S^{ab} [k] = \frac{1}{2} k \; [k], \\
\tilde{S}^{ab} (k) = \frac{1}{2} k \; [k], & \quad \tilde{S}^{ab} [k] = -\frac{1}{2} k \; [k], \quad (A.9)
\end{align*}

which means that we get the same objects back multiplied by the constant $\frac{1}{2} k$ in the case of $S^{ab}$, while $\tilde{S}^{ab}$ multiply $\gamma^{ab} (k)$ by $k$ and $[k]$ by $(-k)$ rather than $(k)$. This
Spin-charge-family theory

also means that when \( ab \) and \( k \) act from the left hand side on a vacuum state \( |\psi_0\rangle \) the obtained states are the eigenvectors of \( S^{ab} \). We further recognize that \( a^a \) transform \( (k) \) into \([k]\), never to \([k]\), while \( a^a \) transform \( (k) \) into \([k]\), never to \([-k]\)

\[
\gamma^a a^b = \eta^{aa} a^b [k], \quad \gamma^b (k) = -i \eta^{ab} a^b [k], \quad \gamma^a [k] = (a^b)^a [k], \quad \gamma^b [k] = -i \eta^{ab} [k].
\]

From Eq. (A.10) it follows

\[
S^{ac} (k)(k) = -\frac{i}{2} \eta^{aa} \eta^{cc} [k][k], \quad \tilde{S}^{ac} (k)(k) = \frac{i}{2} \eta^{aa} \eta^{cc} [k][k],
\]

\[
S^{ac} [k][k] = \frac{i}{2} (k)(k), \quad \tilde{S}^{ac} [k][k] = -\frac{i}{2} (k)(k),
\]

\[
S^{ac} (k)[k] = -\frac{i}{2} \eta^{aa} [k][k], \quad \tilde{S}^{ac} (k)[k] = -\frac{i}{2} \eta^{aa} [k][k],
\]

\[
S^{ac} [k][k] = \frac{i}{2} \eta^{cc} [k][k], \quad \tilde{S}^{ac} [k][k] = \frac{i}{2} \eta^{cc} [k][k].
\]

From Eq. (A.11) we conclude that \( \tilde{S}^{ab} \) generate the equivalent representations with respect to \( S^{ab} \) and opposite.

Let us deduce some useful relations

\[
\begin{align*}
(k)(k) &= 0, \quad (k)(-k) = \eta^{aa} [k], \quad (-k)(k) = \eta^{aa} [k], \quad (-k)(-k) = 0, \\
[k][k] &= [k], \quad [k][k] = 0, \quad [-k][k] = 0, \quad [-k][-k] = 0,
\end{align*}
\]

\[
\begin{align*}
(k) = 0, \quad [k] = 0, \quad (-k)[k] = 0, \quad [-k] = 0, \quad [-k][-k] = 0,
\end{align*}
\]

\[
\begin{align*}
(k) &\neq (k), \quad [k] &\neq [k], \quad (-k) &\neq (k), \quad [-k] &\neq (-k).
\end{align*}
\]

We recognize in Eq. (A.12) the demonstration of the nilpotent and the projector character of the Clifford algebra objects \((k)\) and \([k]\), respectively. Defining

\[
(\pm i) = \frac{1}{2} (\gamma^a \pm \gamma^b), \quad (\pm 1) = \frac{1}{2} (\gamma^a \pm i \gamma^b),
\]

one recognizes that

\[
\begin{align*}
(\pm i) (k)(k) &= 0, \quad (\pm 1) (k)(k) = \eta^{aa} (k), \quad (\pm 1) (k) = i (k), \quad (\pm 1) (k) = 0.
\end{align*}
\]

Recognizing that

\[
(\pm i) (k) = \eta^{aa} (k), \quad (\pm 1) (k) = i (k),
\]

\[
(\pm 1) (k) = 0.
\]
we define a vacuum state $|\psi_0\rangle$ so that one finds
\[
< (k) (k) > = 1, \\
< [k] [k] > = 1. 
\] (A.16)

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for $d$-dimensional space, with $d$ even or odd.

For $d$ even we simply make a starting state as a product of $d/2$, let us say, only nilpotents $ab(k)$, one for each $S_{ab}$ of the Cartan subalgebra elements (Eq.(A.5)), applying it on an (unimportant) vacuum state. For $d$ odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators $S^{ab}$, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

\[
\begin{align*}
&0d \quad 12 \quad 35 \quad d-1 \quad d-2 \quad | \psi_0 \rangle > \\
&| -k_{0d} | -k_{12} | -k_{35} | (k_{d-1} d-2) | \psi_0 \rangle > \\
&\vdots \\
&| -k_{0d} | -k_{12} | -k_{35} | (k_{d-1} d-2) | \psi_0 \rangle > \\
&od \quad 12 \quad 35 \quad d-1 \quad d-2 \quad | \psi_0 \rangle > \\
&\vdots \\
&\vdots 
\end{align*}
\] (A.17)

All the states have the same handedness $\Gamma$, since $\{ \Gamma, S_{ab} \} = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrates that for $d$ even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $(k_{ab})$, by transforming all possible pairs of $(k_{ab})_{mn}$ into $[-k_{ab}][m][-k_{mn}]$. There are $S_{mn}, S^{mn}, S_{mn}, S^{mn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object $\gamma^a$ being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either $d$ even or odd.

While $S^{ab}$ which do not belong to the Cartan subalgebra (Eq. (A.5)) generate all the states of one representation, $\bar{S}_{ab}$ which do not belong to the Cartan subalgebra (Eq. (A.5)) generate the states of $2^{d/2-1}$ equivalent representations.
Making a choice of the Cartan subalgebra set (Eq. (A.5)) of the algebra $S^{ab}$ and $\widetilde{S}^{ab} S^{03}, S^{12}, S^{56}, S^{78}, S^{10}, S^{1112}, S^{1314}, S^{03}, \widetilde{S}^{12}, \widetilde{S}^{56}, \widetilde{S}^{78}, \widetilde{S}^{10}, S^{1112}, S^{1314}$, a left handed ($\Gamma^{(13,1)} = -1$) eigenstate of all the members of the Cartan subalgebra, representing a weak chargeless $u_R$-quark with spin up, hyper charge $(2/3)$ and colour $(1/2, 1/(2\sqrt{3}))$, for example, can be written as

$$\psi = \frac{1}{2} (\gamma^0 - \gamma^3)(\gamma^1 + i \gamma^2)(\gamma^5 + i \gamma^6)(\gamma^7 + i \gamma^8) \psi\psi\psi\psi.$$  \hfill (A.18) 

This state is an eigenstate of all $S^{ab}$ and $\widetilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (A.5)).

The operators $\widetilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (A.5)), generate families from the starting $u_R$ quark, transforming the $u_R$ quark from Eq. (A.18) to the $u_R$ of another family, keeping all of the properties with respect to $S^{ab}$ unchanged. In particular, $\widetilde{S}^{01}$ applied on a right handed $u_R$-quark from Eq. (A.18) generates a state which is again a right handed $u_R$-quark, weak chargeless, with spin up, hyper charge $(2/3)$ and the colour charge $(1/2, 1/(2\sqrt{3}))$

$$\widetilde{S}^{01} (i)(+)(+) | (+)(+) || (+)(-)(-) = -i \frac{1}{2} (i)(+)(+) | (+)(+) || (+)(-)(-).$$ \hfill (A.19)

Below some useful relations are presented

\[ N^\pm_+ = N^\pm_+ \pm i N^\pm_0 = - (\mp i)(\mp), \quad N^\pm_0 = N^\pm_0 \pm i N^\pm_0 = (\pm i)(\pm), \]
\[ \tilde{N}^\pm_+ = - (\mp i)(\mp), \quad \tilde{N}^\pm_0 = (\pm i)(\pm), \]
\[ \tau^{1\pm} = (\mp)(\mp), \quad \tau^{2\mp} = (\mp)(\mp), \]
\[ \tilde{\tau}^{1\pm} = (\mp)(\mp), \quad \tilde{\tau}^{2\mp} = (\mp)(\mp). \] \hfill (A.20)

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