Forward Looking Session

Small and Large Graphs

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Simon Fraser University & IMFM

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Signs of global warming

Observation. When I was young, there was much more snow than today. I remember that the snow level was always well above my head.

Some trends in modern graph theory

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Observation. Today, the mainstream seems to be about large graphs. (Also evident from several talks at this conference.)

Is this a sign of a *global* change?

Main reasons and some examples

- Applications
  - (Theoretical) computers science (e.g. computational complexity, expanders)
  - Emergence of large networks and large data sets
  - Internet graph
  - Social networks
  - Bioinformatics (Evolution trees, genomics, protein folding)
  - Biomedicine (living cells, brain network)
  - Mathematics (e.g. number theory)
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  - Graph limits

Chromatic number

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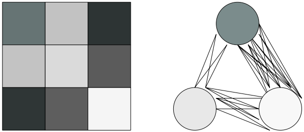
Theorem (Dvorak, Kral, Thomas 2010+). G planar and the distance between any two triangles of G is $\ge 10^{100}$, then G is 3-colorable.

For practical reasons this seems like a result about infinite graphs (or the Grötzsch theorem with at most one triangle allowed).

Regularity lemma and its success

Theorem (Szemeredi Regularity lemma).
$\forall m, \varepsilon > 0 : \exists M = M(m, \varepsilon)$ with the following property:

$\forall G$ with $|G| \geq M$ has an ε-regular partition $V_1, \dots, V_k$, where $m \leq k \leq M$.



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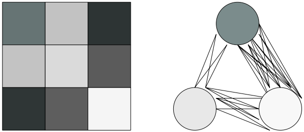

Drawback: M is a tower of exponents of height $O(\varepsilon^{-5})$.
Known lower bounds (Gowers 1997) are also towers of exponents.

Applications in algebra and in extremal combinatoric.

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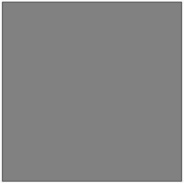
We will need new methods to deal with large graphs. Probabilistic approach has been used throughout.

Graph limits and graphons

Theorem. A large (dense) graph can be approximated by a graphon, a symmetric measurable function $W : [0,1] \times [0,1] \to [0,1]$.

Example: A graphon for $G(n,p)$ satisfies $W(x,y) = p$.



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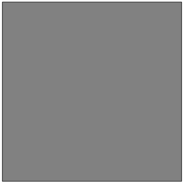

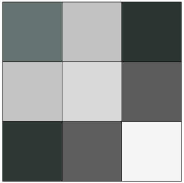

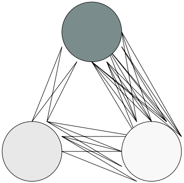

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Thank you . . .