26th Ljubljana-Leoben seminar

Bovec, Slovenia, September 20th-22nd

September 2012
Organized by

FMF, University of Ljubljana
FAMNIT and IAM, University of Primorska

Organizing committee

Norbert Seifter, Primož Potočnik, Aleksander Malnič and Katja Berčič

Keynote speakers

Pablo Spiga, University of Milano-Bicocca
Wolfgang Woess, Graz University of Technology
Programme

Thursday, September 20th

14.50  Welcome
15.00  Gašper Košmrlj, Realizations of the game domination number
15.20  Jernej Rus, Stable Traces
15.40  Sara Zemljič, Sierpiński graphs as spanning subgraphs of Hanoi graphs
16.00  Polona Pavlič, Formulas for various domination numbers of products of paths and cycles
16.20  Coffee break
16.40  Petr Gregor, Subcube isoperimetry and power of coalitions
17.00  Martin Milanič, On graphs whose complement and square are isomorphic
17.20  Boštjan Kuzman, On symmetric tetravalent graphs

Friday, September 21st

9.30  Pablo Spiga, How vertex stabilizers grow?
10.20  Martin Škoviera, Eulerian trails in regular eulerian graphs of odd order
10.40  Coffee break
11.00  Karina Chuda, On S(2, 1)-labeling of generalized Blanuša snarks
11.20  Rok Požar, Sectional split extensions arising from lifts of groups
11.40  Štefko Miklavič, On distance-regular Cayley graphs on abelian groups
12.00  György Kiss, Rainbow-free colorings of PG(n, q)
12.20  Group photo, lunch
14.00  Project and problem sessions
19.30  Conference dinner

Saturday, September 22nd

9.30  Tomaž Pisanski and Nino Bašić, A System for Generating and Dissecting Families of Tessellations
10.20  Robert Jajcay, Girth of a group
10.40  Coffee break
11.00  Wolfgang Woess, On the duality between jump processes on ultrametric spaces and random walks on trees
11.50  Martin Mačaj, Trees T satisfying W(L3(T)) = W(T)
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26th Ljubljana-Leoben seminar on Graph Theory

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How vertex stabilizers grow?

Pablo Spiga, *University of Milano-Bicocca, Italy*
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Here we are interested in highly transitive graphs. There are various natural ways to “measure” the degree of transitivity of a graph and, in this talk, we look at two possibilities. First, we consider graphs $\Gamma$ having a group of automorphisms acting transitively on the paths of length $s \geq 1$, starting at a given vertex. Clearly, the larger the value of $s$ is, the more symmetric the graph will be. However, we show that large values of $s$ impose severe restrictions on the local structure of $\Gamma$ and, sometimes, on the size of the stabilizer of a vertex of $\Gamma$. This will lead us to the second perspective. We take the size of the stabilizer of a vertex of $\Gamma$ as a measure of the transitivity. This measure is somehow unbiased among the graphs having the same number of vertices. Again we present some results showing, in some very specific cases, that nature is not as diverse as one might expect: graphs have either rather small vertex stabilizers or they can be classified. Finally we give some applications of these investigations: to the enumeration problem of vertex transitive graphs and to the problem of creating a database of small vertex transitive graphs. Still, we will have more problems than results.

On the duality between jump processes on ultrametric spaces and random walks on trees

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The purpose of these notes is to clarify the duality between a natural class of jump processes on compact ultrametric spaces – studied in current work of Bendikov, Girgor’yyan and Pittet – and nearest neighbour walks on trees. Processes of this type have appeared in recent work of Bellisard and of Kigami. Every compact ultrametric space arises as the boundary of a locally finite tree. The duality between the random processes arises via the Dirichlet forms: one on the tree associated with a random walk and the other on the boundary of the tree, which is given in terms of the Naim kernel. Here, it is explained that up to a linear time change by a unique constant, there is a one-to-one correspondence between the above processes and Dirichlet regular random walks.
On $S(2,1)$-labeling of generalized Blanuša snarks
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An $S(2,1)$-labeling of a graph is a variation of the usual proper vertex coloring of a graph requiring an additional condition on colors of vertices at distance at most two.

Generalized Blanuša snarks form one of the first known infinite classes of snarks, 3-valent graphs with chromatic index 4.

While no generalized Blanuša snark admits a $S(2,1)$-labeling with 6 colors, each generalized Blanuša snark has one with 8 colors. In this talk, generalized Blanuša snarks admitting no $S(2,1)$-labeling with 7 colors will be characterized.

Subcube isoperimetry and power of coalitions
Petr Gregor, Charles University, Czech Republic
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We determine the minimal number of $d$-dimensional subcubes with a vertex in $A$ and simultaneously a vertex not in $A$, over all sets $A$ of $k$ vertices in the $n$-dimensional hypercube. This extends a classical result of Harper on the edge-isoperimetric problem in the hypercube. We study properties of extremal sets with means of harmonic analysis of corresponding Boolean functions. Applications range from labeling vertices of the hypercube so that the total maximal deviation of labels on subcubes is minimized, to study of influence of coalitions in simple voting games via their Banzhaf power index.

Girth of a group
Robert Jajcay, Comenius University, Slovakia
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We propose a new measure of complexity of finite groups: The girth of a finite group $G$ is the maximal girth of a (connected) Cayley graph $C(G, X)$. Our preliminary results suggest a close correlation between the “abstract complexity” of a group and its girth, where, for example, the girth of a nilpotent group has been shown to be linear in its nilpotency and the girth of a solvable group is exponential in its derived length. Furthermore, the
problem of classifying the girth of specific classes of finite groups is related to their transitive permutation representations. This suggests the necessity to study the girth of full symmetric and alternating groups first.

The presented results have been obtained in collaboration with Prof. Exoo at Indiana State University.

**Rainbow-free colorings of PG(n, q)**

György Kiss, Eötvös Loránd University, Hungary
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A C-hypergraph $\mathcal{H} = (X, \mathcal{C})$ has an underlying vertex set $X$ and a set system $\mathcal{C}$ over $X$. A vertex coloring of $\mathcal{H}$ is a mapping $\phi$ from $X$ to a set of colors $\{1, 2, \ldots, k\}$. A strict rainbow-free $k$-coloring is a mapping $\phi : X \rightarrow \{1, \ldots, k\}$ that uses each of the $k$ colors on at least one vertex such that each $\mathcal{C}$-edge $C \in \mathcal{C}$ has at least two vertices with a common color. If $X_i = \phi^{-1}(i)$, then a different but equivalent view is a color partition $X_1 \cup \cdots \cup X_k = X$ with $k$ nonempty classes. A coloring is called balanced, if $-1 \leq |X_i| - |X_j| \leq 1$ holds for all $i \neq j$.

Let $\Pi$ be an $n$-dimensional projective space and $0 < d < n$ be an integer. Then $\Pi$ may be considered as a hypergraph, whose vertices and hyperedges are the points and the $d$-dimensional subspaces of the space, respectively. In this talk we prove some bounds on the balanced chromatic numbers and present new rainbow-free colorings of projective spaces.

**Realizations of the game domination number**

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Domination game is a game on a finite graph which includes two players. First player, Dominator, tries to dominate a graph in as few moves as possible; meanwhile the second player, Staller, tries to hold him back and delay the end of the game as long as she can. In each move at least one additional vertex has to be dominated. The number of all moves in the game in which Dominator makes the first move and both players play optimally is called the game domination number and is denoted by $\gamma_g$. The total number of moves in a Staller-start game is denoted by $\gamma'_g$. It is known that $|\gamma_g(G) - \gamma'_g(G)| \leq 1$ for any graph $G$. Graph $G$ realizes a pair $(k, l)$ if $\gamma_g(G) = k$ and $\gamma'_g(G) = l$. We present realizations of all possible pairs and show how to extend them.
to be highly connected. We also present the 3/5-conjecture and show some constructions of 3/5-trees.

**On symmetric tetravalent graphs**  
Boštjan Kuzman, *University of Ljubljana and IMFM, Slovenia*  
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We complete a result by C. Praeger and A. Gardiner on 4-valent symmetric graphs by applying the lifting method for elementary-abelian covering projections. In particular, for \( p \neq 2 \), the graphs whose quotient over some \( p \)-elementary abelian group of automorphisms is a cycle, are described in terms of linear codes.

Joint work with A. Malnič and P. Potočnik

**Trees \( T \) satisfying** \( W(L^3(T)) = W(T) \)  
Martin Mačaj, *Comenius University, Slovakia*  
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Let \( G \) be a graph. Denote by \( L^i(G) \) its \( i \)-iterated line graph and denote by \( W(G) \) its Wiener index. We find an infinite class of trees \( T \) satisfying \( W(L^3(T)) = W(T) \), which disproves a conjecture of Dobrynin and Entringer [Electronic Notes in Discrete Math. 22 (2005) 469–475]. This is a joint work with Martin Knor, Primož Potočnik and Riste Škrekovski.

**On distance-regular Cayley graphs on abelian groups**  
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Let \( G \) denote a finite abelian group with identity 1 and let \( S \) denote an inverse-closed subset of \( G \setminus \{1\} \), which generates \( G \) and for which there exists \( s \in S \), such that \( \langle S \setminus \{s, s^{-1}\} \rangle \neq G \). In this talk we give a classification of distance-regular Cayley graphs \( Cay(G; S) \) for such pairs of \( G \) and \( S \).
On graphs whose complement and square are isomorphic
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We study square-complementary graphs, that is, graphs whose complement and square are isomorphic.
We prove several necessary conditions for a graph to be square-complementary, describe ways of building new square-complementary graphs from existing ones, construct infinite families of square-complementary graphs, give a characterization of natural numbers n for which there exists a square-complementary graph of order n, and characterize square-complementary graphs within various graph classes. The bipartite case turns out to be of particular interest.
Joint work with Anders Sune Pedersen and Daniel Pellicer.

Formulas for various domination numbers of products of paths and cycles
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By means of path algebras we present a constant time algorithm for solving different domination problems on the subclass of polygraphs, rotagraphs and fasciagraphs. As these graphs include products (the Cartesian, strong, direct, lexicographic) of paths and cycles, we implement the algorithm to get formulas in the case of the domination numbers, the Roman domination numbers and the independent domination numbers of products of paths and cycles where the size of one factor is fixed, i.e. independently of the size of the second factor. We also show that the values of the investigated graph invariants on the fasciagraphs and the rotagraphs with the same monograph can only differ for a constant value.

A System for Generating and Dissecting Families of Tessellations
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We outline a system for producing and modifying tessellations in the plane, maps on surfaces and polyhedra in space. This is a significant upgrade
of the system that has been developed in the past. The system has useful applications in art and design. The current version of the system is written in Mathematica and is a compilation of several programs that have been written in the past. It covers five overlapping themes: Operations on Tessellations, Counterchanging, Recursive Dissections, Plait-work construction and Understanding the Europa Pattern.

A similar version of this talk was presented earlier this year at the conference “Space, Time, and Matter” in Brioni Islands.

(Work in progress with Godfrey DeWitt, Thomas W. Tucker and Bašić, supported in part by a grant from the Picker Interdisciplinary Science Institute, Colgate University and the GReGAS grant of the European Science Foundation/ Eurocores-EUROGIGA. (ARRS Grant N1-0011)

Sectional split extensions arising from lifts of groups

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In this talk we consider the lifting problem with an additional condition on how a group should lift: given a connected graph \( X \) and a group \( G \) of its automorphisms, find all connected regular covering projections \( \varphi: \tilde{X} \to X \) along which \( G \) lifts as a sectional split extension. By this we mean that there exists a complement \( \tilde{G} \) of the group of covering transformations \( CT(\varphi) \) within the lift \( \tilde{G} \), such that an orbit of \( \tilde{G} \) intersects each fibre in exactly one vertex. As an application, all connected elementary abelian regular covers of the complete graph \( K_4 \) along which a cyclic group of order 4 lifts as a sectional split extension are constructed.

Stable Traces

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At National Institute of Chemistry Slovenia a novel polypeptide self-assembly strategy for nanostructure design was recently presented. We developed the underlying mathematical model. A construction of a polypeptide polyhedron is modelled with a stable trace in the corresponding graph of the polyhedra. Here a stable trace is a double trace with two additional conditions—having no retracing and no repetition through a vertex. It is
proved that the graphs that admit stable traces are precisely graphs with minimum degree 3. Double traces, proper traces, parallel and antiparallel double traces are also introduced. Computational results for several polyhedra are presented.

**Eulerian trails in regular eulerian graphs of odd order**

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Every vertex \( v \) in a connected \( 2d \)-regular graph \( G \) divides any eulerian trail \( T \) into \( d \) closed subtrails \( T_1, T_2, \ldots, T_d \) based at \( v \). If \( G \) has odd order, the number of edges of \( G \) is \( d \) times an odd number, so one may ask whether \( T \) and \( v \) can be chosen in such a way that all the subtrails \( T_i \) have odd length. The answer is trivially positive for \( d = 1 \), and is positive also for \( d = 2 \). We show that for \( d = 3 \) the answer is again positive, and explain why this case is important for determining the flow number of an arbitrary signed eulerian graph.

Our methods can be used to prove results about decompositions of eulerian graphs into two or three closed trails of odd length based at the same vertex under much weaker assumptions.

This is a joint work with Edita Máčajová.

**Sierpiński graphs as spanning subgraphs of Hanoi graphs**

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Hanoi graphs \( H_p^n \) form a natural mathematical model for the Tower of Hanoi game with \( p \) pegs and \( n \) discs. Sierpiński graphs were independently introduced from different studies, our motivation for them is the fact that \( S^n_3 \) is isomorphic to \( H^n_3 \) for any \( n \).

Although for any \( p, n \in \mathbb{N} \), the graphs \( S^n_p \) and \( H^n_p \) are defined on the same vertex set, they cannot be isomorphic anymore for \( p > 3 \) and \( n > 1 \). This follows, for instance, from the fact, that for these values of the parameters \( |E(S^n_p)| < |E(H^n_p)| \). Therefore it is natural to ask whether an isomorphic copy of \( S^n_p \) can be a spanning subgraph of \( H^n_p \). This is true if and only if \( p \) is odd (or trivially if \( n = 1 \)).

This is joint work with Andreas M. Hinz and Sandi Klavžar.
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