On trees satisfying $W(L^3(T)) = W(T)$

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joint work with
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Line graphs

Line graph of a graph G is the graph, denoted by $L(G)$, whose vertices are edges of G, and two vertices of $L(G)$ are adjacent iff they are incident as edges of G.

$L(K_{1,n}) = K_n$, $L(C_n) = C_n$, $L(P_n) = P_{n-1}$.

Iterated line graph $L^{i+1}(G) = L(L^i(G))$.

Wiener index

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Conjecture Dobrynin, Entringer (2005)
For $i \geq 3$ there are no trees satisfying $W(L^i(T)) = W(T)$.

Wiener index

M. Knor, P. Potočnik, R. Škrekovski(2009)

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If T is not a subdivision of $K_{1,3}$ or H_0,
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Subdivisions of H_0

M. Knor, M.M., P. Potočnik, R. Škrekovski(2011)
A subdivision $T = H_{(a,b,c,d,e)}$ of H_0 satisfies $W(L^3(T)) = W(T)$ iff $d = 1$, $e = 2$ and there exist nonnegative integers $i \geq j$ such that

$$\begin{aligned} a &= 128 + 3i^2 + 3j^2 - 3ij + i \\ b &= 128 + 3i^2 + 3j^2 - 3ij + j \\ c &= 128 + 3i^2 + 3j^2 - 3ij + i + j \end{aligned}$$

Some details

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Compute $\Delta = W(L^3(H_{(a,b,c,1,2)})) - W(H_{(a,b,c,1,2)})$.

Some details

$$\Delta = 3(a^2 + b^2 + c^2) - 3(ab + ac + bc) - a - b + c + 128$$

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$$\begin{aligned}\Delta &= 3(a^2+b^2+c^2) - 3(ab+ac+bc) - a - b + c + 128 \\ &= \frac{3}{2}\Big[(a-b)^2 + (c-b)^2 + (c-a)^2\Big] - a - b + c + 128\end{aligned}$$

Some details

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\begin{aligned}
\Delta &= 3(a^2+b^2+c^2) - 3(ab+ac+bc) - a - b + c + 128 \\
&= \frac{3}{2}\Big[(a-b)^2 + (c-b)^2 + (c-a)^2\Big] - a - b + c + 128 \\
&= \frac{3}{2}\Big[(i-j)^2 + i^2 + j^2\Big] - 3i^2 - 3j^2 + 3ij
\end{aligned}
$$

Some details

$$\begin{aligned}\triangle &= 3(a^2+b^2+c^2)-3(ab+ac+bc)-a-b+c+128\\ &= \frac{3}{2}\Big[(a-b)^2+(c-b)^2+(c-a)^2\Big]-a-b+c+128\\ &= \frac{3}{2}\Big[(i-j)^2+i^2+j^2\Big]-3i^2-3j^2+3ij\\ &= 0,\end{aligned}$$

Needle in a haystack

The smallest tree T satisfying $W(L^3(T)) = W(T)$ is $H_{(128,128,128,1,2)}$ on 388 vertices.

It is the unique tree on 388 vertices with this property.

There are approximately $7,5 \cdot 10^{175}$ trees on 388 vertices and

approximately $2,5 \cdot 10^{175}$ trees on 387 vertices.

Thank You